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T — ON THE LUMINOUS EFFICIENCY OF METEORS

by

Franco Verniani

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ON THE LUMINOUS EFFICIENCY OF METEORS

by

Franco Verniani[1]

Summary.--The ratio of the photographic luminous efficiency τ_p to the square of the density ρ_m of about 400 Super-Schmidt meteors, precisely reduced by Jacchia, has been computed directly from the observational data. Fragmentation has been taken into account in the computation.

After critically reviewing previous papers on the subject, the author discusses his assumptions and the procedure he employed. The theory allows the evaluation of the quantity τ_p/ρ_m^2. It is possible to determine τ_p only in the few cases in which the meteors show positive evidence of being of asteroidal origin. The correct dependence τ_p on v is found, after allowing for the different mean densities of meteors in short-period and long-period orbits, since the meteor density is independent of the velocity v. The author shows that a power law $\tau_p = \tau_{op} v^n$ best represents τ_p as a function of v. The detailed behaviour of τ_p may actually be extremely complicated, since most of the meteor light comes from emission lines whose intensity may not vary uniformly or even smoothly with velocity. The exponent n turns out to be 1.0 ± 0.15 for both faint and bright meteors. The best small-camera meteors have been used for comparison. The value of τ_{op} comes out to be about 1×10^{19} zero mag g^{-1} cm^{-3} s^4, which agrees very well with the value inferred by McCrosky and Soberman from the results of artificial meteors. The masses computed at Harvard on the basis of Öpik's original theory therefore appear to be about 6 times smaller than those obtained using the value of the luminous efficiency derived in the present paper.

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[1]On leave: Centro Nazionale per la Fisica dell' Atmosfera del C.N.R., Roma, Italia.

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At a velocity of 40 kms^{-1} the photographic luminous efficiency, expressed in physical units as the ratio of the energy radiated in the blue spectral range to the kinetic energy of the meteoroid, is about 2.10^{-3}. At the same velocity the mass of a zero visual magnitude meteoroid is roughly 0.8 g. The ionizing probability β of an ablated meteor atom can be estimated from τ_p and from some results of Davis and Hall (1963) on combined radio and photographic observations in meteors; at $v = 32\ kms^{-1}$ β turns out to be 0.01.

The analysis shows no dependence of τ_p on mass or brightness and fails to detect any appreciable change of τ_p in the course of a meteor trajectory. Moreover, the results show that τ_p is independent of atmospheric density. Luminous efficiency does seem to depend on fragmentation in the sense that nonfragmenting meteors appear to produce light more efficiently than those that crumble easily. The results obtained for small-camera meteors are in general agreement with those obtained for Super-Schmidt meteors, but it appears that fireballs must have a density about twice that of fainter meteors. The mean relative density of meteors in the main recurrent showers has been also determined. Each shower appears to have its peculiar density. Jacchia obtained a similar result from an analysis of points near the beginning of meteor trajectories.

Introduction

By far the most precise astronomical data for meteor orbits have been obtained by use of photographic techniques (Jacchia and Whipple, 1961). Photographic data also make it possible to evaluate physical quantities, including those that cannot be determined by other means of investigation, such as, for instance, Jacchia's fragmentation index. Such basic quantities as the masses of meteoroids are, however, still very uncertain, although the photographic technique, by providing a light curve, is much more helpful than other means of investigation. The large uncertainty results from the lack of knowledge of the luminous efficiency of meteors. The purpose of this work is to extract, from the Harvard photographic meteors, direct information concerning this parameter.

Since the luminous intensity I of a meteor is generally accepted to be proportional to the kinetic energy of the ablated atoms,* the luminosity equation is usually written in form

$$I = -\frac{1}{2}\,\tau v^2 \frac{dm}{dt} \quad , \tag{1}$$

m and v being the mass and velocity of the body; τ, the luminous efficiency; and $\frac{dm}{dt}$, the rate of the mass loss in atomic (or molecular) form. The gas-cap radiation will be neglected because the meteors that we will refer to, have been photographed in blue emulsion, and there is experimental evidence that such radiation is important only in the red and the infrared regions (Millman and Cook, 1959).

The proportionality between I and $\frac{dm}{dt}$, i.e., between the luminous intensity and the rate of ablation of free atoms, is quite clear. In contrast, the proportionality between I and v^2 seems much harder to understand. It is actually easy to envisage a very intricate dependence of I, i.e., of τ, on the velocity v. Let us consider the spectrum of a meteor with a given velocity. If we progressively increase the velocity, the intensity of the single lines will change; some low-excitation lines will fade out; new lines will appear; and certainly the total emitted light will not be a simple function of the velocity. It is evident, moreover, that the luminous intensity depends on the spectral range that one considers. In the following we will refer to a visual luminous intensity I_v and to a photographic luminous intensity I_p, defined by the sensitivity of the blue emulsion generally used in Harvard photographic studies. Accordingly, we will have two different luminous efficiencies τ_p and τ_v, defined by

$$I_p = -\frac{1}{2}\,\tau_p\, v^2 \frac{dm}{dt} \tag{2}$$

*The photographed spectra of meteors confirm that the kinetic energy lost by deceleration of the body does not contribute to the light.

and

$$I_v = - \frac{1}{2} \tau_v v^2 \frac{dm}{dt} \quad . \tag{2a}$$

If we assume, as is generally accepted, that the dependence of the luminous efficiency on the velocity can be expressed by a power law of the form $\tau \sim v^n$, and if we introduce the color index of the meteors C, which is defined as the difference M_p-M_v between the photographic and the visual magnitudes we get

$$C = - 2.5 \log \frac{I_p}{I_v} = - 2.5 \log \frac{\tau_{op}}{\tau_{ov}} - 2.5 \, (n_p - n_v) \log v \quad . \tag{3}$$

The symbols are self-explanatory.

Jacchia (1957a) found that C is independent of the meteor velocity; this result has been confirmed by the analysis of the complete set of data, precisely reduced by Jacchia, concerning Super-Schmidt material (Jacchia, Verniani and Briggs, unpublished). We therefore have observational evidence that the exponents n_p and n_v are equal--a result that is far from obvious. This being so, in the following n will refer both to n_v and n_p. Jacchia's results show also that C decreases in absolute value with the brightness. Hence the ratio τ_{op}/τ_{ov} must be a function of the brightness and consequently of the mass of the meteors. Knowing the values of C as a function of M_p allows us to determine τ_v as a function of τ_p . For meteors brighter than $M_p = -2$, we can safely assume $C \doteq -1.9$; then $\tau_v = 0.17 \ \tau_p$. For meteors with M_p between -2 and +1.5 Jacchia's results can be approximately described by the linear equation

$$C = 0.28 \ M_p - 1.34 \quad , \tag{4}$$

so that

$$\log \frac{\tau_v}{\tau_p} = 0.11 \ M_p - 0.54 \quad . \tag{5}$$

Since at present only Öpik (1933, 1955) has studied theoretically the luminous efficiency in the visual range, his work has necessarily been the only basis for the computation of meteoric masses. In fact, Whipple (1938) approximated Öpik's results for bright meteors by assuming $\tau_v = \tau_{ov}\, v$ with $\tau_{ov} = 8.5 \times 10^{-10}$ scm^{-1} in the spectrum range between 4500 and 5700 A. All Harvard meteoric masses have been computed with this value and with the linear dependence of τ on v . Both τ_{ov} and the exponent n are very uncertain. Öpik (1958) writes that his basic work of 1933 is "a semi-empirical, semi-theoretical approach on the basis of a combination of classical and quantum-mechanical principle," but Thomas and Whipple (1951) have considered it only "a rough approximation." No attempt will be made here to summarize the work of Öpik; the number of assumptions he introduced to get his results is such that even a rough estimate of the errors involved in the final results is impossible. Only a comparison with direct experimental results can afford such an estimate. In the revision of his own work Öpik (1955) confirmed n = 1 for bright meteors and reduced the value τ_{ov} to about 60 percent of the preceding value. The uncertainty involved in τ_{ov} may, however, be as large as two orders of magnitude (Whipple and Hawkins, 1959). The determination of masses through the motion of meteoric trains led to a value of τ_{ov} about 220 ± 150 times less than Öpik's original value (Cook, 1955). Conversely, the extrapolation of recent results of McCrosky (1963) and McCrosky and Soberman (1962) on artificial meteors suggests an intermediate value between the two extremes, as does a work on a few clearly recognized asteroidal meteors (Cook, Jacchia and McCrosky, 1963). A theoretical study on meteor density (Verniani, 1962) leads to the conclusion that Öpik's value cannot be wrong by an order of magnitude. We will later examine more closely the results obtained with artificial pellets and with the asteroidal meteors in searching for the range in which τ_{op} should lie and for its most probable value.

Current ideas, summarized by McKinley (1961), on the dependence of τ on velocity are that n = 1 applies only to the brighter photographic meteors; for fainter meteors τ should vary more slowly than the first power of v, perhaps even as a negative power of v for fainter dustballs. Although Millman and McKinley (1963) attribute to Whipple the conclusion that for faint meteors τ is almost independent of velocity, the original source is Öpik (1955), who obtained theoretically for dustballs n = - 0.88 and then assumed the round relation $\tau_v = \frac{2000}{v}$ scm^{-1}, and Jacchia (1957b), who for faint Super-Schmidt meteors found τ independent of velocity. This result was obtained from the fitting of the atmospheric profile computed by means of meteor decelerations to the standard atmosphere of that time (ARDC 1956), which had a slope different enough from the present U.S. Standard (1962) to produce a nonnegligible error in n. A re-analysis of the Super-Schmidt meteors, using the new standard atmosphere, shows that n cannot be far from 1 (Jacchia, Verniani, and Briggs, unpublished). Unfortunately, the results are only slightly sensitive to a variation of n. The results of the present work show clearly that τ_p is independent of the brightness of meteors. A dependence of the luminous efficiency on the magnitude or on the mass should not be easy to justify theoretically, although it seems that the spectrum of meteors changes with the brightness in such a way that the emission in the red appears to become important for fainter meteors. Ceplecha (1959) has suggested this spectrum change to explain the results on the color index that he obtained by using panchromatic emulsions, which attain maximum sensitivity for a wavelength equal to 6700 A. Davis' (1961) photoelectric measurements confirmed this suggestion. A variation of τ with brightness was also predicted by Kallmann (1955), who concluded that the luminous efficiency should be proportional to a power of the duration of the individual meteors. Obviously such an hypothesis has absolutely no physical meaning.

Recently Ananthakrishnan (1960, 1961), trying to remove the disagreement between the theoretical beginning and end heights and the measured heights of the photographic meteors reduced by Hawkins and Southworth (1958), introduced the hypothesis that τ_p varies along the trail of an individual meteor in proportion to the atmospheric density ρ_a. Evidently this assumption has no physical basis either. The emitted light must be proportional to ρ_a because of the proportionality between ρ_a and the rate at which atoms leave the body and are responsible for the emission of light. Ananthakrishnan's hypothesis means that the emitted light is proportional to ρ_a^2, and this is absurd; of course the observational results confirm that such an hypothesis is wrong. Incidentally, the above-mentioned discrepancy between the theoretical and the observed heights is without any doubt an effect of fragmentation, as Hawkins and Southworth correctly stated on the basis of Jacchia's (1955) fundamental finding. That the experimental heights of the Super-Schmidt meteors seem to fit the curve computed with $\tau \sim \rho_a v$ better than the theoretical curve based on $\tau \sim v$ is solely the result of the upward shifting of the meteoric heights produced by the fragmentation. This has the same effect as increasing luminous efficiency with ρ_a would. Moreover, when Ananthakrishnan shows that the less-fragmenting part of the Hawkins-Southworth meteors are not so far removed from the theoretical curve as the Draconids of Jacchia, Kopal and Millman (1950), he finds only the very well-known fact that the fragmentation of the members of the Draconid shower is extreme. The fragmentation index χ of Draconids is near to 2, while the average χ for all the Super-Schmidt meteors is 0.25 and that of the group selected in his paper is certainly even smaller.

Ceplecha (1958) and Ceplecha and Padevět (1961) have attempted to evaluate n by using Harvard data on small-camera meteors (Jacchia, 1952) and on Super-Schmidt meteors (Hawkins and Southworth, 1958). Their results are $n = -0.33 \pm 0.61$ for small-camera meteors and $n = -2.8 \pm 0.2$ for Super-Schmidt meteors. These values of n appear unacceptable. There are several reasons for this:

(1) Ceplecha used the theoretical relationship between the intensity at maximum light I_m and the mass outside of the atmosphere m_∞. When there are flares--as there frequently are in small-camera bright meteors--or diffuse fragmentation, as in Super-Schmidt meteors, that relation ceases to be meaningful.

(2) He wrote such a relation, following Levin (1956), in a form very similar to that of Herlofson (1948), which is known to be a rough approximation even in the cases in which the theory could be successfully used (Verniani, 1961).

(3) In the same equation Ceplecha eliminated the mass m_∞ by making use of another equation of Levin (1956), obtained with the assumption that the velocity of the meteoroid does not change during the entire path. Moreover, that equation contains the atmospheric density of the end point of the meteor. Such a value is often meaningless because of the very irregular behavior of meteors in the last part of their recorded path, as a result of crumbling and fragmentation (Jacchia, 1955). It may differ from the corresponding theoretical value by an order of magnitude.

(4) The Harvard small-camera material, especially the earlier Massachusetts meteors, is very inhomogeneous with regard to the accuracy of the data. Therefore a good analysis can be done not only by rejecting those meteors for which the time of appearance is poorly determined or the mass is not accurately known, but also by using a suitable system of weights, as Jacchia did in his 1952 analysis, for taking into account the accuracy of the computed decelerations. This accuracy varies remarkably from one meteor to another.

(5) Ceplecha used the Rocket Panel atmosphere, which now is known to be in error both in slope and in density values at meteoric heights.

(6) He found n by plotting a certain computed quantity versus log v_∞. The theoretical slope of the expected straight line is $9 + n$. Even if the other strong causes of error were absent, the small value of n in comparison with 9 clearly makes his determination highly unreliable, because of the nonnegligible fluctuations of the computed quantities caused by (a) experimental errors; (b) variations in the drag coefficient with height and velocity; (c) irregularities in the shape of the meteoroids; and (d) fluctuations from the standard atmosphere.

(7) The masses of Hawkins-Southworth meteors are averages of individual values given by each of the two plates of the same meteor and computed with relationship (48) of Hawkins' (1957) paper on the method of reduction of short-trail meteors. Often there was a large difference between the two individual values. Although on the average those masses represent a fair approximation, they are much less precise than the masses Jacchia computed for small-camera meteors and for his selected sample of long-trail Super-Schmidt meteors by integrating the light curve (Jacchia, physical data not yet published). This difference can be another reason why the exponent Ceplecha found for Super-Schmidt faint meteors is so much further off the mark than that of small-camera meteors. The principal reason is the basically different degree of fragmentation among the two samples of meteors.

Levin (1956) made a statistical verification of the theory of meteors by writing $I_m \sim m_\infty^\alpha v_\infty^\beta \cos^\gamma Z_R$ (Z_R = zenith angle of the meteor path) and by computing α, β and γ by least squares for Harvard small-camera data. The masses Levin used were those computed by Jacchia on the basis of $\tau_v \sim v$; therefore, according to single-body theory, the value of β must be 4. The result, $\beta = 3.98 \pm 0.27$ and $\alpha = 1.00 \pm 0.05$, confirmed that for bright meteors the classic theory is adequate to describe accurately the average behavior of the phenomenon. On the other hand the computed value of γ was completely different from the expected value of 1. Levin and Majeva (1963) did the same kind of verification on the Super-Schmidt meteors reduced by Hawkins and Southworth, but the result was quite different. They divided those meteors into two groups, according to the different degree of fragmentation estimated from the beginning and end heights. The group of less-fragmenting meteors gave $\beta = 3.67 \pm 0.08$, while the other group, containing meteors that experienced more severe fragmentation, gave $\beta = 2.67 \pm 0.10$. These deviations from theory can be easily understood with the aid of some of the preceding remarks on Ceplecha's work. They show that one cannot extract any information on n by following that route. As a curiosity, in the above-mentioned book Levin, by using the masses computed with Öpik's value of τ_{ov}, determines the average visual magnitude of a meteor having $m_\infty = 1$ g, $v_\infty = 10$ km/s^{-1}. Then, making use of this value, he reverses his procedure to find the value of τ_{ov}. It is peculiar that the result he gets is completely different from Öpik's value.

I shall conclude this introduction with some brief remarks on the units in which the luminous intensity (and consequently the luminous efficiency) is expressed. In many papers the luminous intensity is measured in ergs; in this case the relation between the visual magnitude M_v of a meteor and the corresponding intensity I_v has been given by Öpik (1958) as

$$M_v = 24.30 - 2.5 \log I_v \quad , \tag{6}$$

where the constant 24.30 is determined by taking into account the sensitivity curve of the human eye; it corresponds to the sun's stellar magnitude -26.72 and to the energy distribution of the solar radiation. This system, in which τ is a dimensionless quantity, seems quite natural to the physicist, but it is not convenient for practical use, especially in photographic work. Therefore Jacchia (1948) introduced a system that takes as unit of intensity that of a star of magnitude zero. In this system τ has dimensions zero mag $g^{-1}cm^{-2}s^{3}$. In the following we will use Jacchia's system, which has the clear advantage of being closer to the observed quantities, i.e., to the magnitudes. In Jacchia's system we have

$$M_p = -2.5 \log I_p \tag{7}$$

and

$$M_v = -2.5 \log I_v \quad . \tag{7a}$$

From equations (6) and (7a) it follows immediately that the logarithmic difference between τ_v expressed in cgs and in Jacchia's units is 9.72. Therefore we have

$$(\log \tau_{ov})_{\ddot{O}pik} = -9.07 \qquad \text{(cgs)}$$

$$(\log \tau_{ov})_{\ddot{O}pik} = -18.79^{*} \text{ (Jacchia's units)} \; .$$

For bright meteors, to which a color-index correction -1.8 is applicable, we have

$$(\log \tau_{op})_{\ddot{O}pik} = -18.07 \quad .$$

In addition to Jacchia's units for I_p and τ_p, we will use cgs units for all other quantities.

*The value used in Harvard's work is -18.91. The difference of 0.12 results from the different form of eq. (6), which was previously established by Öpik (1937) as: $M_v = 24.6 - 2.5 \log I_v$.

Procedure used for determining n

The drag equation is usually written in the form

$$a = \gamma \, A \, \rho_m^{-2/3} m^{-1/3} \rho_a \, v^2 \quad , \tag{8}$$

where a is the deceleration of the meteor; ρ_m, its density; γ, the drag coefficient; and A, the shape factor. The integration of equation (2) yields

$$m = 2 \int_t^{t_E} \frac{I_p}{\tau_p v^2} \, dt \quad , \tag{9}$$

t_E being the time at which the meteor ends. We assume that the terminal mass can be neglected; this should be correct in most cases (Jacchia, 1948; Verniani, 1959). Equation (9) may be written

$$m \, \tau_p = \frac{2}{\bar{v}^2} \int_t^{t_E} I_p \, dt = \frac{2E}{\bar{v}^2} \quad , \tag{10}$$

where $E = \int_t^{t_E} I_p dt$ is the integrated brightness, and $\bar{v}$ is a value of v at some instant between t and t_E. The symbol τ_p refers to the same velocity. Eventually, by eliminating m between equations (8) and (10), we obtain

$$\frac{\tau_p}{\rho_m^2} = \frac{2E}{\bar{v}^2} \left(\frac{a}{\gamma \, A \, \rho_a v^2} \right)^3 \quad . \tag{11}$$

In writing equation (11), we admit the equality of the "dynamic" mass derived from the drag equation and of the "photometric" mass obtained by integrating the light curve. Such an assumption is valid only for meteors that ablate without fragmenting. Therefore, for most of the Super-Schmidt meteors, which crumble during their flight in the atmosphere, such an assumption is not right. It is well known that the photometric mass takes into account the light emitted from all fragments and therefore may also be considered reliable for fragmenting meteors. In contrast, the dynamic mass corresponds only to the mass of the larger fragments to which the measured deceleration refers. We must conclude that in the general case, when fragmentation is present, equation (11) overestimates τ_p/ρ_m^2. We will see in section b) under "Data and results", however, that it is always possible to make use of equation (11) by taking into account the effects of fragmentation by means of Jacchia's fragmentation index χ.

The photographic data allow the determination of the velocity v, deceleration a, integrated brightness E and height z. The U. S. Standard Atmosphere (1962) gives ρ_a as function of z. The Super-Schmidt meteors have several decelerations each; we will see that the most reliable for our purpose are those near to the beginning of the light. Therefore, for each meteor only one value $\bar{v}$ has been computed. We assumed $\bar{v}$ to be equal to the value of v corresponding to $m = \frac{1}{2} m_\infty$, i.e., to the value zero of Jacchia's mass-loss parameter s. Inspection of the light curves and of the observational velocity curves shows that $v_{s=0}$ represents a very adequate approximation of $\bar{v}$. The difference between $\bar{v}$ and v_∞ is generally very small. It is of the order of 1 or 2 percent for slow meteors and goes down to 0.1 or 0.2 percent for the fast ones.

The shape factor A is unknown. According to Öpik (1958), rotation, vibration and oscillation of meteors during their flight smooth their shape so that we can ignore the possible variations of A during the meteor life and accept the value $A = \left(\frac{9\pi}{16}\right)^{1/3} = 1.21$ corresponding to a spherical shape. This value is usually employed in Harvard's work. The effect of fragmentation makes useless any search for a better approximation. We shall discuss this effect later.

The last quantity to be considered is the drag coefficient γ. This quantity is not well known. The results yielded by an interpolation formula worked out by Baker (1959) on the basis of the theory of Baker and Charwat (1958) for the drag coefficient in the transitional flow region do not agree with recent experimental results (Maslach and Schaaf, 1962). Baker's formula shifts the transition from the free molecular flow to continuum flow to heights greater than the reality. Cook (1963, private communication) has given another expression for γ in the form

$$\gamma = 0.55 \left[1 + 8.3 \times 10^{-5} (r\rho_a)^{-\frac{1}{2}}\right] , \qquad (12)$$

where r, the equivalent radius of the pellet, and ρ_a are expressed in cgs units. This equation applies only to compact bodies, like meteorites and asteroidal meteors.* The use of equation (12) for the Super-Schmidt meteors shows that many of these are still in free molecular flow, although the hypothesis that these meteors are single compact bodies is certainly wrong for most of them. For better consideration of the effect of fragmentation, we will assume that all the Super-Schmidt meteors are in free molecular flow. We will therefore use $\gamma = \gamma_F = 1.1$, the value generally assumed for satellites in free molecular flow (Cook 1959; Jacchia 1963). This value is also used in the general analysis of the physical data of the Super-Schmidt meteors (Jacchia, Verniani and Briggs, unpublished).

If we make the extreme assumption that equation (12) may also be applied to the Super-Schmidt meteors, considered as single compact bodies, we obtain from the observational data a value of n only slightly smaller than that obtained by assuming $\gamma = \gamma_F$. The difference is about 15 percent, which is very reassuring, since it means that if γ is not exactly what we have chosen, the reliability of the final result is not affected.

The data employed for the analysis are those of the 413 Super-Schmidt meteors precisely reduced by Jacchia (astronomical data published by Jacchia and Whipple, 1961; physical data not yet published). The old Harvard small-camera data have been used separately, although their accuracy and homogeneity is much poorer than that of the Super-Schmidt, to provide a basis of comparison for larger masses and lower heights. For these meteors, which are generally very bright, we have used Cook's expression (12) for γ.

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*In eq. (12) γ tends to infinity when $\rho_a \to 0$. Therefore, when eq. (12) yields a value greater than $\gamma_F = 1.1$, it is understood that γ must be taken equal to γ_F

The quantities involved in the computation of τ_p/ρ_m^2 are variously affected by observational errors. The velocities and the heights are quite reliable. The error in the velocity for about half of the Super-Schmidt meteors is of the order of 0.1 percent. The small-camera meteors are also generally accurate in v to better than 1 percent. The decelerations, on the other hand, are not so reliable; the probable error may differ widely from meteor to meteor. This is particularly true for small-camera meteors, whose probable errors range from 0.01 to 10 times the value of the deceleration itself. Also, the reliability of the probable error varies greatly, depending on the number of breaks used to compute the deceleration. The integrated brightness is generally more reliable when t is near the beginning time t_B, because of the uncertainty in the interpolation to zero of the intensity curve at the end of the detectable trajectory. The lower reliability of the values of τ_p/ρ_m^2 computed from data near the end of the trajectory is augmented by the effects of fragmentation, which affects equation (8) by increasing the cross-sectional area A. Hence it is clear that, to do a correct analysis, we must use a suitable system of weights, which will take all these factors into account. In his analysis of the small-camera data for determining the atmospheric density as a function of the height Jacchia (1952) solved the same problem. Therefore I have used here a slightly modified form of the weight function introduced by Jacchia. This modified formula has been also used in the general analysis of the Super-Schmidt physical data, which will be published soon.

For all meteors the quantity $\gamma A\rho_a v^2 a^{-1}$ (proportional to $\rho_m r_\infty$) has been computed from each deceleration. Then a weighted average has been taken for every meteor, by using a weight p defined as

$$p = 10\psi\left(\frac{a}{\text{p.e.}}\right)\times\psi(N-3)\times\left(\frac{m}{m_\infty}\right)^{\frac{1}{2}} \quad , \qquad (13)$$

when

$$\psi(x) = \tfrac{1}{2}\left[1 + \text{erf}\,(2.5\log x - 2.1)\right] \quad ; \qquad (14)$$

p.e. is the probable error in the deceleration a; and N is the number of shutter breaks used in the least-squares solution (which contains 3 unknowns) for determining a. Then, by means of the weighted average values of $\gamma A\rho_a v^2 a^{-1}$, I have computed τ_p/ρ_m^2 for each meteor to which a weight $w = (\Sigma p)^{\frac{1}{2}}$ has been assigned for the final analysis. I have put w into this form because, since the results obtained for meteors with many decelerations are somewhat more reliable than those obtained for meteors with only two decelerations, we must attribute a larger weight to them. Taking $w = \Sigma p$ would overweight the slow meteors, which are generally longer, and the final analysis would be biased. Calculations with different weights have been done to investigate how much the chosen system of weights affects the final results. The result was that the analysis of the Super-Schmidt meteors is only slightly affected by the system of weights. This was to be expected because of the general reliability and homogeneity of Jacchia's Super-Schmidt material. For small-camera meteors, the inaccuracy in the determination of the deceleration varies the weights p from less than 0.01 to about 9, and the choice of the weight system is, conversely, very important.

The quantity τ_p/ρ_m^2 varies with the velocity $\bar{v}$ of the meteor. We shall attribute this variation uniquely to τ, i.e., we shall assume that the meteor density ρ_m does not depend systematically on the velocity. Of course there is no reason why the meteor density should vary continuously with the velocity. Nevertheless, we must consider two different circumstances that, if ignored, would affect the conclusions, perhaps remarkably. The first is the possibility that some among the very slow meteors may be of asteroidal origin; in this case the value of τ_p/ρ_m^2 in the region of the lowest velocities would be smaller and our determination of n overestimated. It is easy enough to obviate this difficulty by taking out of the analysis all the meteors that show a difference from the average larger than the expected scatter in the values. Naturally, some meteors will be clearly suspected of having different densities, but it is a difficult task to decide whether or not certain others should be excluded.

The second circumstance to keep in mind is Jacchia's (1958) discovery of the difference in fragmentability between the meteors of the Jupiter family and the long-period meteors. This difference leads us to think that the different fragmentation may correspond to a difference in density between the two groups. This hypothesis is corroborated by the fact that long-period meteors begin to appear at a height that on the average is about 4 km greater than that of short-period meteors (Jacchia, 1963). Therefore the sporadic meteors have been divided in two groups according to their aphelion distance in order to search for a difference in the averages of the τ_p/ρ_m^2. Such a difference has actually been found, and we will see that taking it into account changes n appreciably, because one group is concentrated toward the low velocities and the other, toward the high velocities. Obviously this difference could be caused either by a variation of τ_p resulting from differences in chemical composition or by different densities or by a variation of both these quantities. However, the well-established current ideas on the cometary origin of meteors and on Whipple's (1950, 1951) icy-comet model lead us to conclude that the difference must be ascribed to the density only. In fact, according to Whipple's theory, short-period comets evaporate faster, which means that short-period meteors should come from the inner core of their parent comets, while the long-period meteors should come from more external layers, subjected during the comet's life to a smaller pressure. We can therefore expect that short-period meteors have a greater solidity than long-period ones. Thus it is easy to understand a difference in density among meteors with much different orbital characteristics, while it is difficult to find reasons for justifying a difference in composition.

We should expect a nonsystematic spread in the values of the density ρ_m among sporadic meteors. Taken together with the irregularity in the shape of the bodies, the observational errors and the effects of fragmentation, which will be discussed in detail later, this spread must give a wide scatter in the individual values of τ_p/ρ_m^2. In spite of this scatter, all of whose causes are random and unpredictable, we may expect to arrive at correct conclusions by taking averages of τ_p/ρ_m^2 in groups with a large number of meteors in the same range of velocities or better, with a least-squares solution for all the data.

Data and results

a) The value of n.--For the determination of n by the least-squares method it is convenient to put the dependence of τ_p on v in a logarithmic* form:

$$\log \frac{\tau_p}{\rho_m^2} = \log \frac{\tau_{op}}{\rho_m^2} + n \log v \quad , \tag{15}$$

where v is $\bar{v}$. Of the 413 original Super-Schmidt meteors, 12 are not available for our purpose because some basic physical data are missing or because their precision is lower. Taking out all the meteors having a clearly anomalous $\log \tau_p/\rho_m^2$ leads us to eliminate from the analysis 40 meteors (10 percent of the total number), 36 of which are sporadic. Thus the analysis concerns 361 meteors, 247 sporadic and 114 shower. The least-squares method

*In the following, log always indicates decimal logarithms.

gives for the sporadic meteors $n = 1.52 \pm 0.15$. This result, however, is not to be considered final because of the difference in density between long-period meteors and those of the Jupiter family. Table 1* contains the uncorrected average values of log τ_p/ρ_m^2 for groups of increasing velocity. These values are given only as a reference for comparison with the final data. Before correcting the data, we computed two other least-squares solutions for estimating how much the chosen form of the weight w could affect the analysis. Two extreme cases have been considered. In the first, all the weights w have been put equal to 1; in the other, individual decelerations instead of individual meteors have been used. The results are both quite close to $n = 1.5$, which confirms that the definition of the weight w affects neither the final results nor their reliability.

Let us now examine how to introduce a correction for the difference in density between meteors of the Jupiter family and long-period meteors. By dividing the sporadic meteors into two groups, one having the aphelion distance $Q < 7$ a.u. and the other having $Q > 7$ a.u., we get the results listed in table 2 and plotted in figure 1. These results show a mean difference of about 0.30 between the values of log τ_p/ρ_m^2 for the two groups. This means that the average density of the short-period meteors is about 1.4 times the density of long-period meteors. The reduction of the values of log τ_p/ρ_m^2 to the same velocity has been done with successive approximations, which lead to a value of n close to 1. If we compute two separate least-squares solutions for the two groups, we do not obtain reliable results. The reasons for this are very clear: (a) The range in velocity becomes too small for both groups; and (b) the statistical fluctuations of the data become important, because the amount of data is diminished. These fluctuations are clearly illustrated in figure 1, which shows that they are particularly important for the long-period meteors, which have a very large scatter. If we remove the extreme-velocity groups from the small-aphelion-distance meteors, we get a slope very close to $n = 1$. Figure 1 also illustrates that the slope $n = 1$ does not contradict the very irregular trend of the long-period meteors. We could correct the results by reducing the average values of velocity groups to the density of the meteors with $Q < 7$ a.u. and in this way get the final value of n. In order to study the residuals, however, it is better also to correct the individual data by subtracting 0.30 from all the values of log τ_p/ρ_m^2 belonging to meteors with $Q > 7$ a.u.

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*In table 1, as in the following tables, the numerical values are sometimes given, for mathematical purposes only, with one digit more than those really significant.

After the reduction of all meteors to the average density of the group of the Jupiter family, the least-squares method gives

$$n = 1.01 \pm 0.15 \quad ; \tag{16}$$

$$\log \frac{\tau_{op}}{\rho_m^2} = -17.80 \pm 0.98 \quad . \tag{17}$$

The error in τ_{op}/ρ_m^2 is quite large, because of the large extrapolation involved from the meteor velocities' range to $v = 1\ \mathrm{cms}^{-1}$. The actual error in τ_p in the range 10-70 kms^{-1} in which we are interested is, however, small enough. The least error in the determination of $\log \tau_p/\rho_m^2$ is that corresponding to the centroid of the distribution, for which we have

$$\log \tau_p/\rho_m^2 = -11.281 \pm 0.030; \quad \log v = 6.433 \pm 0.008 \ . \tag{18}$$

For each other point, the error ϵ_v should be given by

$$\epsilon_v = 0.03 + 0.15 \times |\log v - 6.433| \quad . \tag{19}$$

In the following, however, it is convenient to adopt $n = 1.0$, so that the error in $\log \tau_p/\rho_m^2$ will be constant and equal to 0.03. Equation (17) now reads

$$\log \frac{\tau_{op}}{\rho_m^2} = -17.71 \pm 0.03 \quad . \tag{17a}$$

By sorting the sporadic meteors into groups according to their velocity and computing the mean values of $\log \tau_p/\rho_m^2$, we obtain the results shown in tables 3 and 4 and plotted in figures 2 and 3. Since the residuals do not show any other definite dependence on velocity, the power-law $\tau_p = \tau_{op} v^n$ is sufficiently close to reality. It is important to remember that the individual intensity in each wavelength is a function of velocity and that our τ_p corresponds to integrated light, which must depend on v in some complicated, almost certainly irregular manner. In order to have a check on the correction for the Q effect on the values of log τ_p/ρ_m^2, we have arranged 10 equally populated groups of sporadic meteors in order of increasing Q and have computed the averages of the residuals for each of them. The correlation coefficient between the average residuals and Q is -0.28, which shows a practical absence of correlation, because we are dealing with averages and not with individual values for which the large scatter could lower the correlation coefficient. When Q increases, the residuals tend to pass from negative to positive values for meteors with $Q < 7$ a.u., but this cannot affect the results in any way. In fact, for meteors of the Jupiter family, there is no correlation between v and Q.

Figure 4 shows the average values of log τ_p/ρ_m^2 for each shower; the same data are listed with more details in table 5. As I have said before, it is unlikely that different showers have appreciably different composition to account for variations in τ_p; conversely, their different ages and distances from the sun may account for differences in density. Accordingly, the differences $(\log \tau_p/\rho_m^2)_{shower} - (\log \tau_p/\rho_m^2)_{sporadic,\ v = v_{shower}}$ are attributed only to a difference in density. With this assumption the ratios, contained in table 5, of the density of each shower to the average density of the sporadic meteors with $Q < 7$ a.u. have been computed. Northern and Southern Taurids, Southern ι-Aquarids and δ-Aquarids, Perseids, Lyrids and σ-Hydrids have density values close to that of the sporadic meteors of the Jupiter family. Conversely, Quadrantids, Orionids, κ-Cygnids and α-Capricornids have about the same density as the sporadic long-period meteors. As the reader can see from the indicative values of Q listed in table 5, however, some of the showers with $Q > 7$ a.u. have approximately the same density as the sporadic meteors with $Q < 7$ a.u. and vice versa. The members of the Geminid shower have a density four times larger than that of the sporadic meteors. This confirms a result that Jacchia obtained in his earlier work. The extremely low density of Draconids is also confirmed, but the figures in that case do not have much meaning because the effects of fragmentation are too large to be properly taken into account. The results confirm that each shower has its own peculiarities, as Jacchia found from his analysis of points near the beginning of meteors.

b) Effects of fragmentation.--Let us now discuss the effects of fragmentation on the computed values of log τ_p/ρ_m^2. Jacchia (1955) found that the anomalously large increases of the observed decelerations of the Super-Schmidt meteors are explained by progressive fragmentation. He introduced, as a measure of the phenomenon, the fragmentation index χ, defined as

$$\chi = \frac{d}{ds} \log \frac{a_{obs}}{a_T} , \tag{20}$$

where a_{obs} is the observed deceleration (a in the preceding equations); a_T is the deceleration computed from the drag equation by using the photometric mass; and s is the mass-loss parameter, defined as

$$s = \log\left(\frac{m_\infty}{m} - 1\right) . \tag{21}$$

The fragmentation index χ is not easy to determine because it involves the second-time derivative of the velocity. Since χ is, however, the quantity that best describes the fragmentation, it is very helpful for correcting data. Its meaning lies in its being approximately constant for each meteor during the detectable part of its flight. If fragmentation were absent we would find, in the limits of observational error, the same value of τ_p/ρ_m^2 for whatever point of the trajectory the decelerations introduced in equation (11) refer to. But the fragmentation introduces a variation in the computed values of τ_p/ρ_m^2. In fact, by using equations (11) and (20) we can easily find the relation between the values of log τ_p/ρ_m^2 computed in two different points of the trajectory corresponding to s and s_o:

$$\left(\log \tau_p/\rho_m^2\right)_s = \left(\log \tau_p/\rho_m^2\right)_{s_o} + 3\chi\,(s-s_o) . \tag{22}$$

We can immediately see the order of magnitude of the variation introduced by the fragmentation if we consider a meteor for which several decelerations have been determined and for which $\chi = 0.25$, the mean value for sporadic meteors. The difference Δs between the value of s corresponding to the first and to the last determination of the deceleration is about 1. Therefore the fragmentation changes $\log \tau_p/\rho_m^2$ by 0.75. In case of larger χ the effect is, of course, larger. It is clear that all the present data should be reduced to the value of s corresponding to the beginning of fragmentation. Unfortunately, this value is not known, but we could refer the values of $\log \tau_p/\rho_m^2$ to a value of s near the beginning of the meteors. This value may be found by calibrating the results by means of the nonfragmenting meteors. This is not necessary for the purpose of this work, because the use of the weights p, which attribute greater importance to the decelerations in the first part of the trail, when fragmentation has not yet played its role, also satisfactorily accomplishes this task. In fact, for the purpose of empirically finding out the effect of fragmentation on the actual weighted values of $\log \tau_p/\rho_m^2$, all the sporadic meteors with $|\chi| < 0.2$ have been selected and processed by the least-squares method. The solution, after the usual correction for Q, is $n = 1.24 \pm 0.22$ and $\log \tau_p/\rho_m^2 = -11.214 \pm 0.040$ for $\log v = 6.421 \pm 0.011$. The mean values of $\log \tau_p/\rho_m^2$ as functions of velocity are listed in table 6 and plotted in figure 3 for comparison with the results of all the sporadic meteors. The value 1.24 for n agrees, within the limits of the error, with the value 1.01 afforded by the general solution; it is worth noting that if we remove from the general solution the slowest meteors ($10 < v < 15$ kms^{-1}), for which the average value of $\log \tau_p/\rho_m^2$ is smaller by about 0.3 than the expected one,* we would get a slightly smaller value of n. As a compromise between this value and that afforded by the low-fragmentation meteors, $n = 1$ is really the best value for n. The agreement for the value of $\log \tau_p/\rho_m^2$ in the centroid of the distribution is actually very good. For $\log v = 6.42$ we get -11.213 and -11.268. Although we might expect a larger difference and in the opposite direction, as we shall see, τ_p depends on the degree of fragmentation, insofar as meteors that do not fragment seem to be more efficient in producing light. Anyway, the consistency of these results shows the reliability of Jacchia's weight system, which was used throughout.

*This is because of the presence in the group of some meteors that, although they do not clearly have the characteristics of the asteroidal meteors, appear to have a larger density. We cannot even exclude the possibility that the luminous efficiency descreases more rapidly than $\tau \sim v$ for meteors slower than 15 kms^{-1} (Jacchia, 1949). This last possibility seems, however, to be very unlikely.

We must remark that there are cases in which neither χ nor the weight system is sufficient to find out the correct value of log τ_p/ρ_m^2 by using equation (11). This is true when a meteor breaks up into N_F fragments of approximately equal mass before it is detected or before the earliest deceleration is measured. This is the case of the so-called "abrupt-beginning meteors," which appear suddenly at or very near the maximum light, and of the very short meteors, whose light curve is practically reduced only to a flare. In these cases the value of log τ_p/ρ_m^2 is overestimated, and it is easy to see that the difference between the correct and the computed value is log N_F. Fortunately, the number of these kinds of meteors is small, and most of them are among those rejected from the analysis because of the clearly anomalous value of log τ_p/ρ_m^2. Consequently, the reliability of the results does not seem jeopardized by the effects of fragmentation.

It is very important to note that the broad scatter among the individual values of log τ_p/ρ_m^2 is not the result of the fragmentation. In fact, the average deviation of an individual value from the least-squares general solution is 0.53, while the corresponding average deviation for low-fragmentation meteors is 0.51. The average $|\chi|$ for this last group is 0.08; the average individual difference from the average value of $\bar{s} \sim 0.2$,

$$\left(\bar{s} = \sum_i^{1\ldots\nu} p_i s_i \Big/ \sum_i^{1\ldots\nu} p_i \qquad \nu = \text{number of decelerations of each meteor} \right).$$

Therefore the average value of $3\chi\Delta s$ is 0.05, i.e., one tenth of the actual scatter. Even taking into account the relative inaccuracy with which the χ's are known, the discrepancy remains very large. Several factors may account for such a large spread, among which is the existence of groups of sporadic meteors having different density. On the other hand, the mean of the average deviations of the individual values of log τ_p/ρ_m^2 for shower meteors is about 0.4. This shows clearly that the difference in density is not the only cause of the scatter. To explain it, we must remember that the actual error in the values of the deceleration is much larger than the internal error previously considered.

As a matter of fact, the decelerations of this sample of Super-Schmidt meteors have another source of uncertainty: the flutter that affected the rotation of the camera shutters. When the trail covered two or more cycles of the flutter, it was possible to eliminate its effect fairly well, but for shorter trails a correction was impossible (Whipple and Jacchia, 1957). Moreover, the elongated and fuzzy form of the dashes, particularly when wake or blending or both together are present, makes it impossible to know exactly to what the deceleration refers. It can never be too much emphasized that very often in meteor physics the quantities determined from the observations do not correspond strictly with those that enter the equations of the theory.

Another source of error in τ_p/ρ_m^2 is the atmospheric density. We should not forget that local transient fluctuations of the atmospheric density may appreciably affect the results, since τ_p/ρ_m^2 is inversely proportional to the third power of ρ_a.

c) Investigation of the dependence of τ on mass.--Let us now examine the problem of the dependence of the luminous efficiency on the mass of meteoroids. By assuming that τ_p depends on some power P of the initial mass, we can write

$$\tau_p = K_1 \, v^{n_1} \, m_\infty^P \quad . \tag{23}$$

By using equation (2) we get:

$$\tau_p = K \, v^{\frac{n_1 - 2P}{1 + P}} \, E_\infty^{\frac{P}{1 + P}} \quad , \tag{24}$$

where $\frac{n_1 - 2P}{1 + P} = n$. Equation (24) shows that we can investigate the dependence of τ_p on m_∞ by studying the correlation between τ_p and the total integrated brightness E_∞ . This is true even if we assume that τ_p is proportional to the power P of the instantaneous mass. Therefore Jacchia's Super-Schmidt sporadic meteors have been arranged in three equally populated groups in order of increasing ε_∞ ($\varepsilon_\infty = \log E_\infty$). The results, which are listed in table 7 clearly show that τ_p does not depend on the meteor mass. We cannot check whether the exponent n keeps the same value when the brightness changes by using the least-squares method for these three groups, because each group corresponds to a different mean velocity and the actual spread in v becomes too small. Only the intermediate group has roughly the same velocity distribution over the entire sample. Most of the meteors that belong to the faintest group are very slow, while the brightest group contains a great part of the fast meteors. It is possible, however, to arrange the sporadic meteors in three groups of different brightness but of the same distribution in velocity. The results obtained with the least-squares method show that n decreases as we go from the faint meteors to the bright ones. This result disagrees with the current opinions of McKinley (1961), quoted above, and with Öpik's theory, according to which n should decrease with brightness. The differences between n = 1, as found for all the sporadic meteors together, and the single values corresponding to each group of brightness could, however, hardly be said to be significant, since they are of the same order as the probable error involved in these determinations. Moreover, the average deviations of one individual value of log τ_p/ρ_m^2 from the least-squares straight lines are just as large as that found in the general solution involving all the sporadic meteors. Study of figure 5, in which average values of log τ_p/ρ_m^2 taken in intervals of 10 kms^{-1} for each group are plotted, clearly shows that the variations of n given by the least-squares method certainly result more from accidental fluctuations than from a real difference in the behavior of τ_p as function of v. But the final proof of the independence of n from the brightness is afforded by the analysis of the small-camera meteors. These meteors are much brighter than the Super-Schmidt meteors, since their average E_∞ is about 100 times the average E_∞ of the Super-Schmidt meteors. But the value of the exponent n of the small-camera meteors turns out to be very close to 1, as we will see later.

The possibility of a dependence of τ_p on the mass m_∞ has also been studied by means of the residuals Δ , defined as the difference between the values of log τ_p/ρ_m^2 obtained by the observational data and those obtained by the least-squares solution. Table 8 reports the average values of Δ for 10 groups of 25 meteors in order of increasing brightness. These results are also plotted in figure 6. It is clear once more that τ_p does not depend on the mass.

d) The luminous efficiency as a function of fragmentation.--The results of section b) above lead us to investigate whether there is some correlation between the luminous efficiency and the degree of fragmentation of meteoroids. According to their fragmentation index χ , sporadic meteors have been sorted into 10 groups, and the weighted averages of the residuals Δ have been computed. The results are listed in table 9 and plotted in figure 7. A weak correlation appears between Δ and χ . The correlation coefficient is -0.73. An empirical equation correlating $\bar{\Delta}$ and $\bar{\chi}$, obtained by the least-squares method, is

$$\bar{\Delta} = (0.11 \pm 0.04) - (0.50 \pm 0.11)\bar{\chi} \quad . \qquad (25)$$

One would expect that meteors with a large χ had lower density. If this were so, their log τ_p/ρ_m^2 should be larger than the average. The results show just the opposite: $\bar{\Delta}$ is positive for meteors with low χ and negative for meteors with a high degree of fragmentation. Therefore equation (25) may be interpreted as an actual dependence of the luminous efficiency on the degree of fragmentation of the meteors, such that a compact body is more efficient in producing light than a porous, crumbling one. The possible decrease of the density with increasing χ tends to mask the effect, so that the value 4 of the ratio of the luminous efficiencies at the two extremes has to be considered as underestimated. It is worth noting that the average value of $\bar{s}$ does not vary systematically with χ , so that even if we reduce the residuals to the average values of $\bar{s}$, we arrive at practically the same results. The same is true also if we take the data uncorrected for the difference in density between short- and long-period meteors.

An explanation of the dependence of τ_p on χ could be found in the different density of the vapors in the meteoric coma. Low-fragmentation meteors should have a denser coma and radiation arising from the collisions among the molecules of the coma itself. Conversely, the radiation of the most crumbling meteors should be caused by collisions between air molecules and ablated meteor atoms. On the basis of such an explanation we should, however, also expect some dependence of τ_p on the mass. We could also explain the dependence by a difference of composition among the sporadic meteors such that low-fragmentation meteors contain a larger percentage of materials having a higher radiative efficiency than crumbling meteors do. This does not, however, seem very likely, as I pointed out for shower meteors in an earlier section. I must also remark that the residuals Δ are strongly affected by all the causes of error previously mentioned (observational errors, fluctuations in the atmosphere, irregularities in the shape, differences in density of the meteoroids, and so on) and that some error is also present in the values of χ , as is shown by the number of meteors having $\chi < 0$. Therefore the validity of equation (25) must be considered with caution.

A clearer correlation also exists between $\bar{\Delta}$ and $\overline{\log \sigma}$, σ being the coefficient of the mass equation

$$\frac{\dot{m}}{m} = \sigma \, v \, \dot{v} \quad . \qquad (26)$$

The results, contained in table 10, are plotted in figure 8. The regression line is

$$\bar{\Delta} = (-7.03 \pm 1.45) - (0.63 \pm 0.13)\ \overline{\log \sigma}\ , \qquad (27)$$

and the correlation coefficient is -0.75. Also in this case the correlation is not appreciably changed by reducing the residuals to the average value of $\bar{s}$, because $\bar{s}$ does not vary systematically with $\overline{\log \sigma}$. This correlation is similar to the preceding one between $\bar{\Delta}$ and $\bar{\chi}$. In fact, the most efficiently radiating meteors are those with smaller values of σ, i.e., values near to those pertaining to compact bodies. This correlation must, however, also be taken with caution, for substantially the same reasons advanced for the preceding one. Log σ is known with a better accuracy than χ, but its physical meaning is questionable for fragmenting meteors.

e) <u>Independence of the luminous efficiency from the atmospheric density</u>.--As I stated in the introduction, there are no theoretical reasons for a proportionality between the luminous efficiency τ_p and the atmospheric density ρ_a. This can also be proved directly by the observational data. In fact, for a large minority of the Super-Schmidt meteors of the present sample, several decelerations were determined at heights that are enough different to allow us to see if τ_p depends on ρ_a and how. Obviously, when dealing with meteors showing a nonnegligible degree of fragmentation, one must apply a correction $3\chi(s-s_o)$, as discussed in b) above. In table 11 are listed values of log τ_p/ρ_m^2 computed by individual decelerations at different heights for each of several randomly chosen sporadic meteors having χ close to or equal to zero. The table also contains the data concerning a few other meteors that experienced fragmentation in varying degrees, to the extent of showing in real cases how the correction $3\chi(s-s_o)$ works. As we expected, there is absolutely no dependence of τ_p on ρ_a, and the hypothesis advanced by Ananthakrishnan is completely meaningless.

Very recently Rajchl (1963) has found that the luminosity coefficient τ_{op} decreases during the meteor flight, being proportional to $(1 + \rho_a v)^{-1}$. Rajchl reached this conclusion by analyzing only 8 meteors and by using some approximated equations to which we may apply the same criticisms as to Ceplecha's work in point (2) of the introduction. The present results show that this dependence also does not exist.

f) Results for small-camera meteors.--While shower meteors are less than 1/3 of all Super-Schmidt material, they are the majority among small-camera meteors; therefore the number of sporadic meteors available for the present analysis is small. This is also the result of the lack of accuracy in the decelerations of many of them, which often leads to weights smaller than 1. Because of their poor reliability, meteors having $w < 1$ have been removed from the final analysis. Therefore, only 44 sporadic meteors are available for the analysis. The basic results are listed in table 12. The difference in density between long-period meteors and those of the Jupiter family is also present among these meteors, as is shown in the table. The consequent difference in the average values of $\log \tau_p/\rho_m^2$ is estimated to be 0.25 (for Super-Schmidt meteors it was 0.30). After the correction, the least-squares solution gives

$$n = 0.9 \pm 0.5 \quad , \qquad (28)$$

and the centroid of the distribution is

$$\log \tau_p/\rho_m^2 = -11.69 \pm 0.08 \text{ for } \log v = 6.39 \pm 0.02 \quad . \qquad (29)$$

The accuracy of the determined value of n is very poor, but this value is very close to that obtained for the Super-Schmidt meteors. For $\log v = 6.39$ the value of $\log \tau_p/\rho_m^2$ for Super-Schmidt meteors is -11.32 ± 0.04, with a difference of 0.37 ± 0.12 from the value (29) for small-camera meteors. By dividing the sporadic small-camera meteors into two groups, so that the first group contains meteors having ϵ_∞ not exceeding that of the brightest Super-Schmidt meteor, and reducing the results to $\log v = 6.39$, we get:

$$\log \tau_p/\rho_m^2 = -11.16 \pm 0.12 \; ; \; \epsilon_\infty = 0.6 \pm 0.1 \qquad (30)$$

$$\log \tau_p/\rho_m^2 = -11.86 \pm 0.11 \; ; \; \epsilon_\infty = 1.8 \pm 0.1 \quad . \qquad (31)$$

The difference between the two groups is quite clear. The faint meteors have about the same value of log τ_p/ρ_m^2 as the Super-Schmidt (the difference is equal to the probable error), while the bright meteors appear much more dense. The result is

$$(\bar{\rho}_m)_{\text{bright meteors}} \simeq 2(\bar{\rho}_m)_{\text{faint meteors}} \quad . \tag{32}$$

The group of fainter small-camera meteors has an average brightness near to that of the Super-Schmidt meteors of the last of the 10 groups in table 8, which do not show any difference from the fainter Super-Schmidt meteors. The maximum ϵ_∞ for the sporadic Super-Schmidt meteors is 0.94. Therefore we are led to conclude that the gap in density is close to $\epsilon_\infty = 1$. It corresponds roughly to a photographic magnitude -2.7 or to a visual magnitude -1, which is just the limit of the fireballs.

g) <u>The value of τ_{op}</u>.--The observational data allow the determination of the ratio τ_p/ρ_m^2, but it is impossible to separate these two parameters except in the few cases in which we have complete evidence that the meteors are of asteroidal origin. Only one of the discussed samples, meteor no. 1242, has been recognized to be certainly asteroidal by Cook, Jacchia and McCrosky (1963), who found evidence that its composition was similar to that of meteoritic stone ($\rho_m = 3.4$ g cm^{-3}). This meteor has 4 decelerations that allow values of log τ_p/ρ_m^2 very close one another. The weighted average is

$$\log \tau_p/\rho_m^2 = -14.115 \pm 0.076 \text{ for } \log v = 6.037 \quad . \tag{33}$$

The meteor does not show progressive fragmentation. Its value of χ is -0.10, so that a reduction of all the decelerations to the same value of s does not change the value of equation (33). The accuracy of the decelerations is very good. The average weight p is 5.9, and w is about 5. By assuming $\rho_m = 3.4$ g cm^{-3} and n = 1.0, we get

$$\log \tau_{op} = -19.09 \pm 0.08 \quad . \tag{34}$$

Although the quality of meteor no. 1242 is very good, it is obviously unsafe to rely on a value obtained from only one meteor. Fortunately the recent experiences with artificial meteors have led to other estimates of τ_{op}. Let us examine briefly the results of these works. McCrosky (1961) reports the results of the firing of two shaped charges from an Aerobee rocket at a height of 80 km. The liners inserted in the explosive were made of aluminum. From laboratory evidence McCrosky assumed that 2 percent (0.11 g) of the mass was ejected with an initial speed of 14.2 kms^{-1}. From the observed light curve, assuming the linear dependence of τ on v given by Öpik for bright meteors, he found

$$\tau_{ov(Al)} = 9.10^{-10}\ \mathrm{scm}^{-1} \quad , \tag{35}$$

which seems in perfect agreement with the Öpik's earlier value ($8.5 \times 10^{-10}\mathrm{scm}^{-1}$). Of course these two values cannot be compared directly, since aluminum is not a major constituent of meteors. Although the correction cannot be accurately computed, McCrosky was able to work out a lower limit for τ_{ov}. He figured out that a maximum of 83 percent of the radiation could be produced by the oxidation of aluminum. Moreover, he assumed that iron and aluminum had equal efficiency per atom and that all the meteor radiation is caused by the iron, which, following Öpik (1958), he estimated to be 15 percent of the meteor material. The final result was

$$\tau_{ov} = 1.10^{-11}\ \mathrm{scm}^{-1} \quad , \tag{36}$$

85 times less than Öpik's value and about three times larger than Cook's value estimated on the basis of the motion of meteoric trains. Since McCrosky used a color-index correction -1.8 for passing from photographic to visual magnitude, we can go back to τ_{op}, which in Jacchia's units turns out to be

$$\log \tau_{op} = -20.0 \quad . \tag{37}$$

Since this value is considered an extreme lower limit, McCrosky warns that a value 100 times larger (-18.0) is not precluded by the results of his experiment.

More recently McCrosky and Soberman (1962) have produced an iron meteor at a height of about 70 km by accelerating to a velocity of about 10 kms^{-1} a stainless-steel pellet ($m \simeq 2.2$ g) by an air-cavity charge attached to the nose of a seven-stage Trailblazer rocket. The primary purpose of the experiment was to determine the meteor luminous efficiency. The result is

$$(\tau_{op})_{stainless\ steel} = 8.10^{-19} \ \ 0 \text{ Mag } g^{-1} cm^{-3} s^{4} \quad . \qquad (38)$$

The estimated error involved is such that we may write, in logarithmic form,

$$(\log \tau_{op})_{stainless\ steel} = -18.10 \pm 0.1 \quad , \qquad (39)$$

which represents a very accurate result. Three corrections are necessary, however, to convert the value of equation (38) to a luminosity coefficient valid for meteoric material. The first of these is caused by the presence in the pellet of about 20 percent Cr. McCrosky and Soberman conclude that the luminous efficiency of Cr should be roughly equal to that of iron, so that the value of equation (38) should also be true for iron. They warn, however, that the probable extremes are -12 and -12.54. The second correction results from the possibility that the pellet is not completely ablated at the end of the detected luminosity. Fortunately, after a detailed analysis of the ablation experienced by the pellet, McCrosky and Soberman find that no significant correction is necessary for terminal mass in their case.

The last and most important correction is for the composition of meteoroids. There is observational evidence from the spectra of low-velocity meteors that practically all the light seen on a blue emulsion is caused by iron emission. Therefore the luminous efficiency of iron would give that of meteoric material if the iron content in meteoroids were known. Although the composition of iron and stony meteorites is well known, no truly reliable information is available on the composition of cometary meteoroids, which are by far the most common among meteors. McCrosky's extreme estimates are -20.10 and -18.80, which correspond to relative Fe abundances of 1 percent and of 20 percent, with the warning that these should be lower limits because other elements will contribute, though little, to the luminosity. It is not unreasonable at the present stage to assume that the iron percentage in cometary meteors is equal to that of meteoritic stone, i.e., 15.4 percent (Öpik, 1958). In this case the result is

$$\log \tau_{op} = -18.91 \quad . \qquad (40)$$

A detailed discussion for three asteroidal meteors, one iron and two stone, has been done by Cook, Jacchia and McCrosky (1961). The density of these meteors being known, they were able to find the value of τ_{op}: $\log \tau_{op} = -18.37$ for stone. They removed the discrepancy from the value of equation (40) by adopting for the three meteors $\gamma A = 0.92$ instead of $\gamma A = 0.6$. If we assume that the radiation of AlO is negligible for meteors, the value (37a) becomes -19.23, which agrees fairly well with the preceding results. In any case, the reliability of this value has to be considered much lower than that obtained with the Trailblazer. The value of equation (40) has also been accepted by Whipple (1963) in his recent paper on meteoroids and penetration.

By putting the value of equation (40) into equation (17a), we find the average density of the Super-Schmidt sporadic meteors with $Q < 7$ a.u. The result,

$$\bar{\rho}_m = 0.25 \text{ gcm}^{-3} \quad , \tag{41}$$

is in good agreement with the value obtained theoretically by Whipple (1955) on the basis of his well-known cometary model. He found, in fact, that $\bar{\rho}_m$ must lie between 0.2 and 0.4 gcm^{-3}. If we reverse the procedure and assume $\bar{\rho}_m = 0.3 \pm 0.1$ gcm^{-3}, we get from equation (17a) $\log \tau_{op} = -18.76$ with the extreme values -18.51 and -19.11.

Another experiment with a Trailblazer (McCrosky, 1963, private communication) has confirmed the previous result, thus enforcing the reliability of the method. It seems that the efficiency of Cr in producing light is larger than that of Fe. In this case the value of equation (40) should be slightly decreased. In view of the uncertainties still present, the author proposes as a best estimate for $\log \tau_{op}$ the round value $\log \tau_{op} = -19.0$. The masses of meteoroids must therefore be computed by using

$$\tau_p = 1.10^{-19} v \quad , \tag{42}$$

with v expressed in cms^{-1} and I_p in units of zero magnitude. The error in the mass resulting from the intrinsic error of τ_{op} may be estimated to be of the order of 2 or even larger. From the preceding discussion it is clear that it is more likely that equation (42) gives overestimated values of τ_p than vice versa. The error in relative masses, caused by the uncertainty in the adopted value of n, between 10 and 70 kms^{-1} does not exceed 20 percent.

If we express the luminous efficiency τ_p in physical units as the ratio between the energy radiated in the blue-emulsion spectral range and the kinetic energy of the parent meteoroid, we have

$$\tau_p = 5.25 \times 10^{-10} \, v \quad . \tag{43}$$

Therefore τ_p turns out to be 0.002 for a velocity of 40 km/s.

It is interesting to observe that the dependence of τ_p on the velocity appears to be linear, as Öpik predicted in his earlier work (1933) for bright meteors. The linear dependence is, however, also valid for fainter meteors, and here Öpik's theory fails. The present best estimate of the **coefficient** of the luminous efficiency τ_{op} is about 8 times smaller than Öpik's 1933 value* and only 5 times smaller than the value he has proposed in 1955.

On the basis of equation (42) we can evaluate the mass of a zero visual magnitude meteoroid at a given velocity. After converting visual into photographic magnitude by means of equation (4), we may use the following formula, empirically derived by Jacchia (1958) from photographic meteors with smooth light curves:

$$E_\infty = \frac{10^6 \, I_{pm}^{1.093}}{v \, \cos Z_R} \quad , \tag{44}$$

where I_{pm} is the photographic intensity at maximum light; and Z_R is the zenith angle of the meteor path. The integrated brightness E_∞ and the meteor mass outside the atmosphere m_∞ are related [see equation (10)] by

$$m_\infty = \frac{2 \, E_\infty}{\tau_{op} v^3} \quad . \tag{45}$$

At $v = 40 \text{ kms}^{-1}$ and $\cos Z_R = 0.6$ (mean value of $\cos Z_R$ for the Super-Schmidt meteors) m_∞ is 0.84 grams.

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*Harvard meteoric masses are, however, only 6.5 times smaller than the values computed on the basis of eq. (42). See footnote, page 9.

h) <u>The ionizing efficiency</u>.--It appears that at present the uncertainty concerning the ionizing efficiency β of meteors is an order of magnitude larger than the uncertainty affecting the luminous efficiency τ_p. It therefore seems reasonable to deduce the ionizing efficiency from the present results on τ_p and from experimental data. As a result of their program of combined radio and photographic observations of meteors, Davis and Hall (1963) were able to determine the ratio I_p/q for 7 meteors, q being the electronic line density of the meteor trail. Their result was $\log I_p/q = -3.96 \pm 0.14$ at $v = 32.2$ kms^{-1}.

The ionizing efficiency is defined by an equation analogous to equation (1):

$$I_q = -\frac{1}{2}\,\tau_q\,v^2\,\frac{dm}{dt} \quad , \tag{46}$$

I_q being the power going into the production of electrons. The electronic line density is given by

$$q = -\,\frac{\beta}{\mu v}\;\frac{dm}{dt} \quad , \tag{47}$$

β being the probability of ionization of an ablated meteor atom of mass μ. The terms β and τ_q are related by

$$\tau_q = \frac{2\Phi}{\mu v^2}\,\beta \;, \tag{48}$$

where Φ is the mean ionization potential; $\Phi \sim 7$ ev (Cook and Millman, 1955).

From equations (2) and (46), we have

$$\frac{I_p}{q} = \frac{\tau_p}{\beta}\,v^3\,\frac{\mu}{2} \quad . \tag{49}$$

The average mass μ of a meteor atom is 3.8×10^{-23} g (Öpik, 1958). Therefore, at $v = 32.2$ kms^{-1}, we obtain

$$\beta = 0.010; \qquad \tau_q = 5.6 \times 10^{-4} \quad .$$

The errors in τ_p and I_p/q cause an uncertainty in β of a factor of 3. The values of τ_q and β deduced by Davis and Hall were based on Öpik's late evaluation of the luminous efficiency and are consequently about three times larger.

A preliminary result of a study in progress on the dependence on velocity of the ionizing efficiency suggests $\beta \sim v^4$. Accordingly, we have, in cgs,

$$\beta = 9.5 \cdot 10^{-29} \, v^4 \quad .$$

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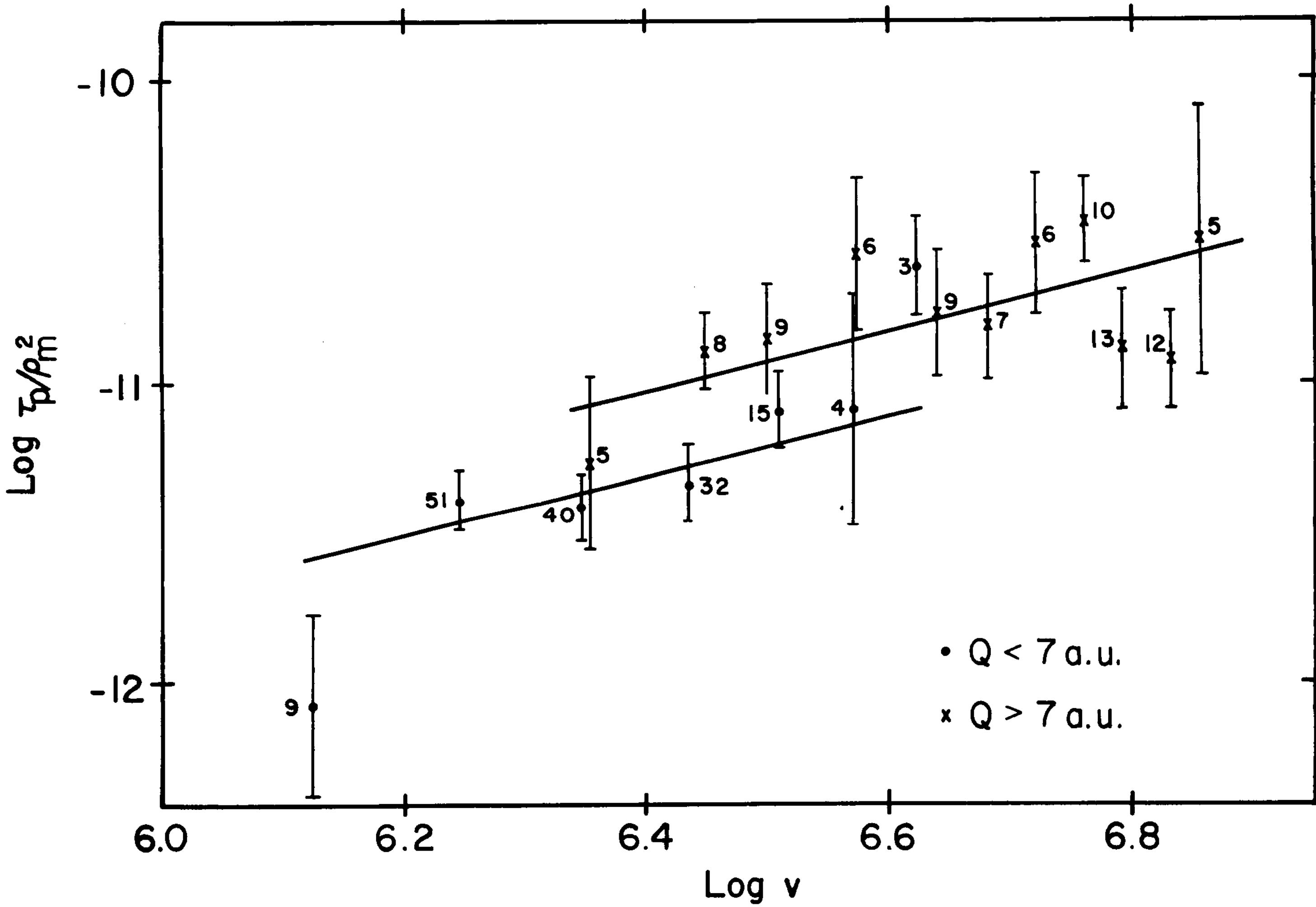

Figure 1.--Super-Schmidt meteors: log τ_p/ρ_m^2 as a function of meteor velocity for meteors in short-period ($Q < 7$ a.u.) and long period ($Q > 7$ a.u.) orbits. The confidence marks represent standard deviations of the means. The number of meteors of each group is written near the points. See table 2.

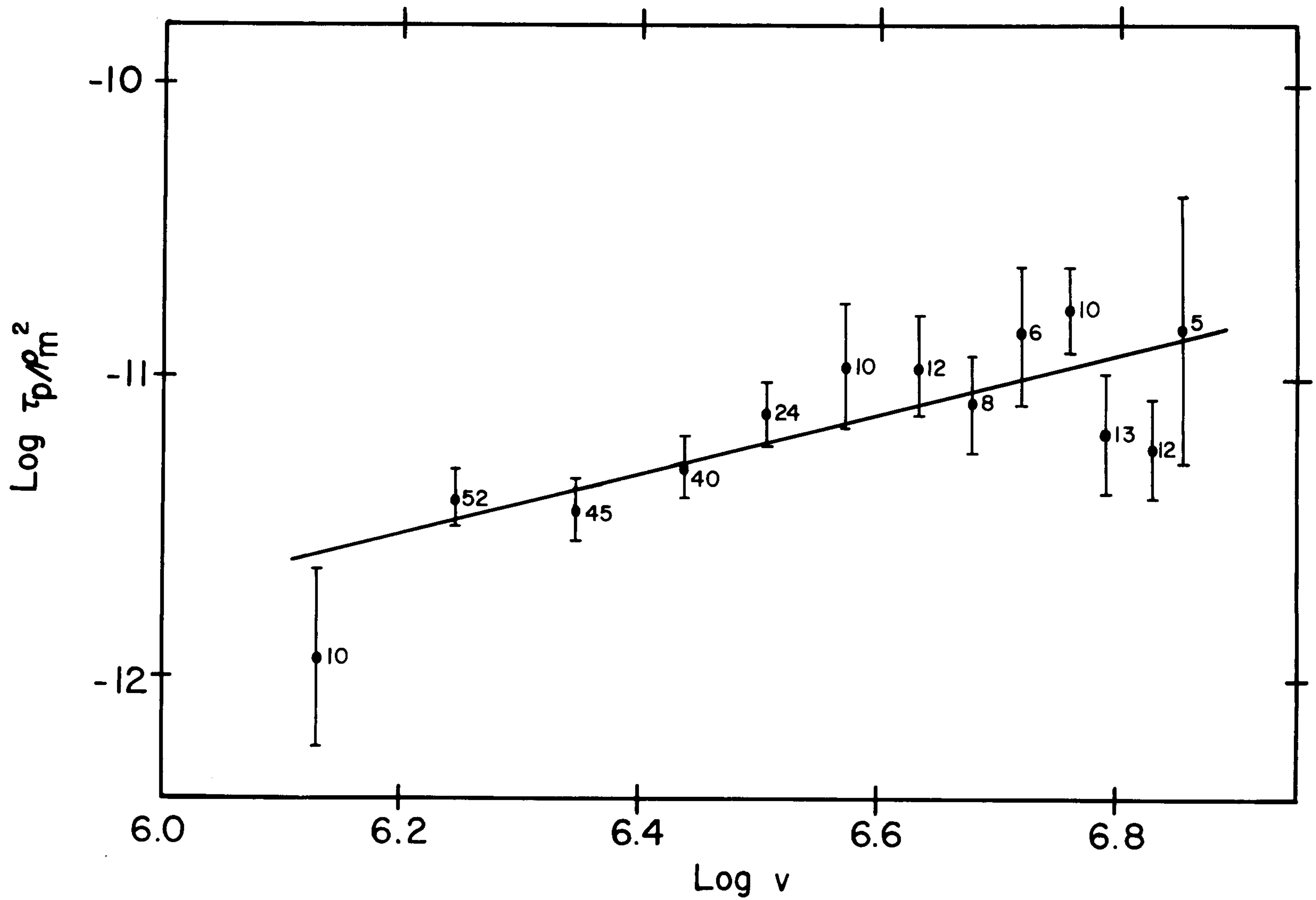

Figure 2.--Sporadic Super-Schmidt meteors: log τ_p/ρ_m^2 as a function of meteor velocity. See table 4.

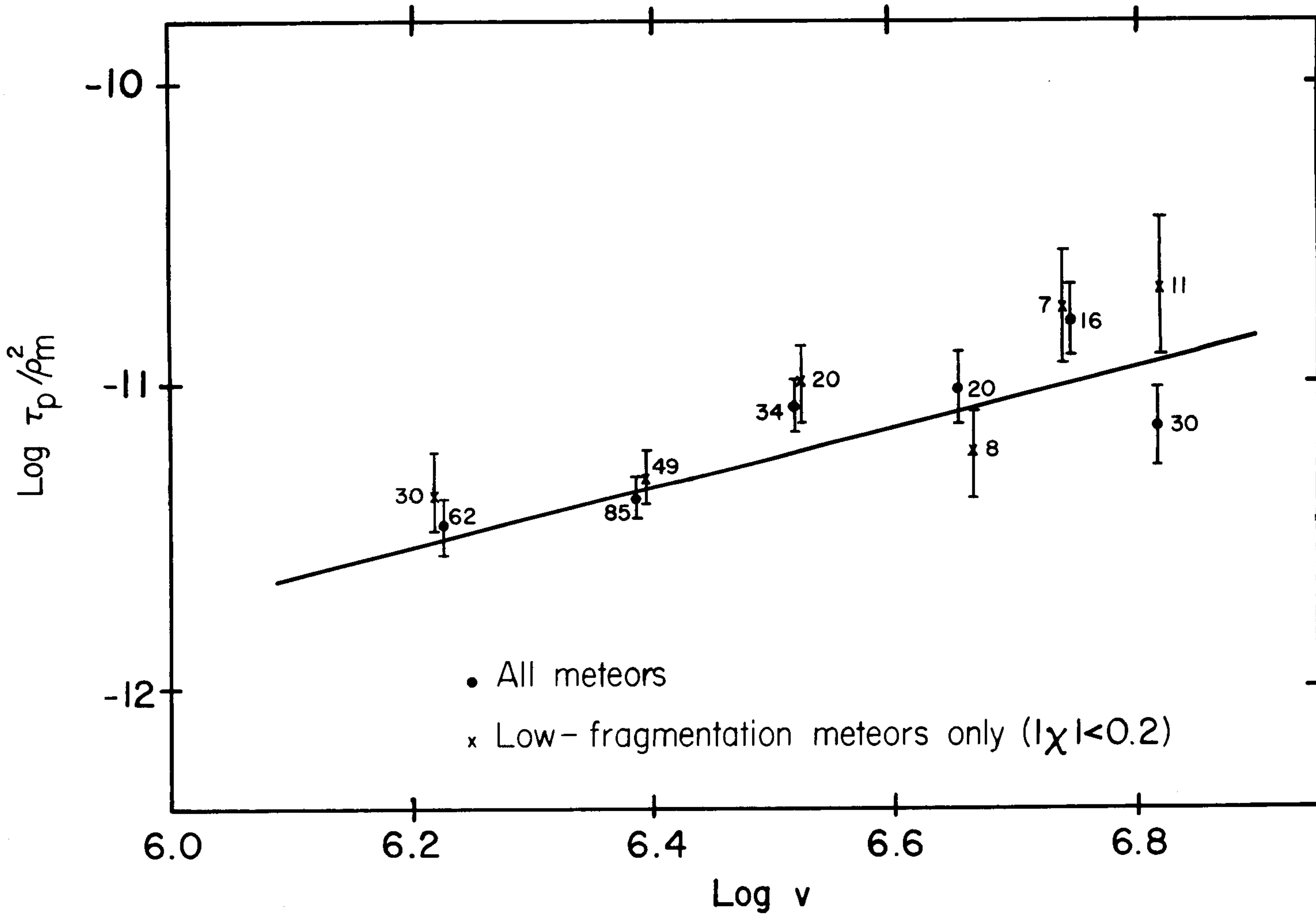

Figure 3.--Sporadic Super-Schmidt meteors: log τ_p/ρ_m^2 vs. log v for all meteors and for meteors having $|\chi| < 0.2$ only. See tables 3 and 6.

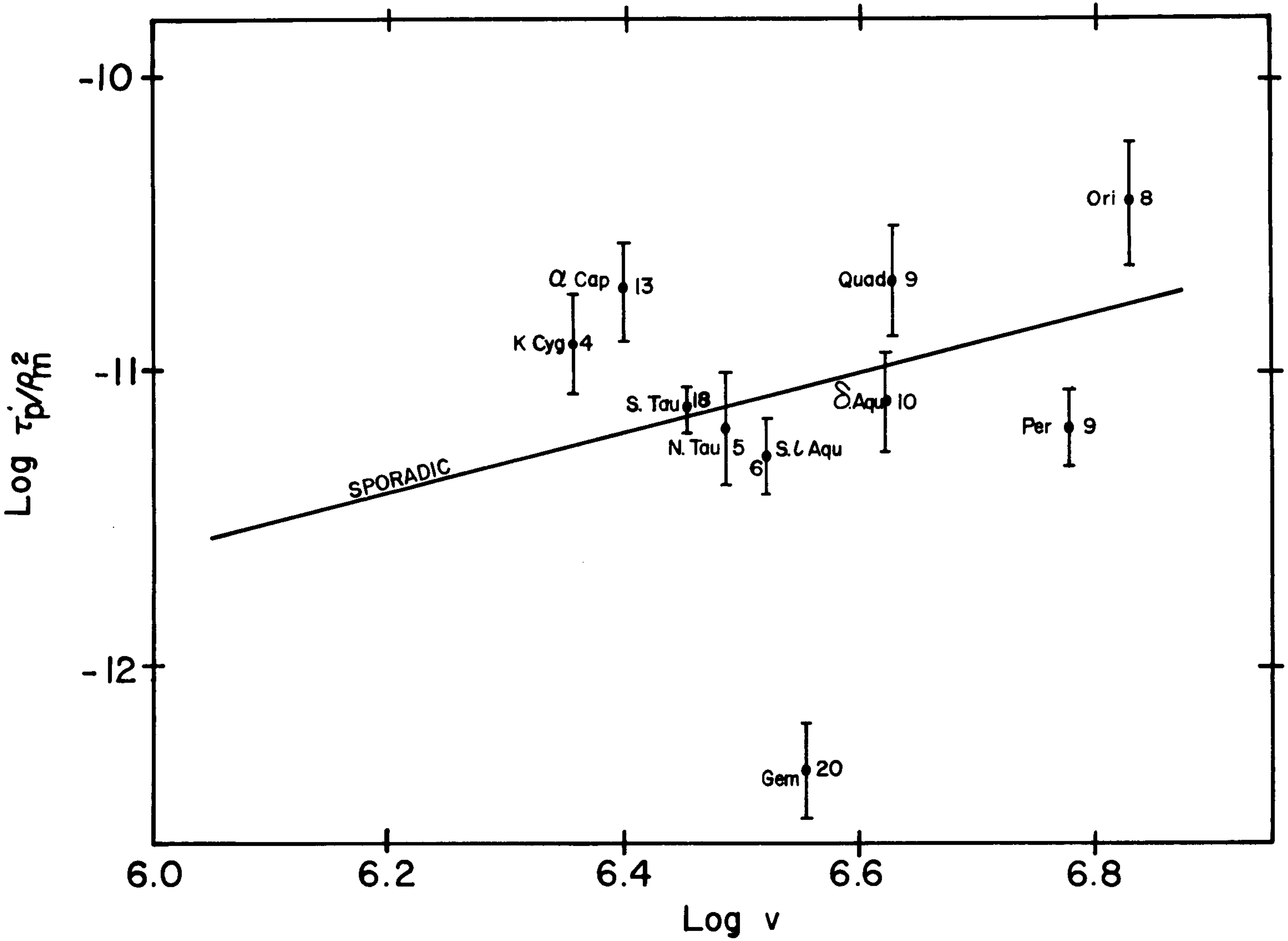

Figure 4.--Super-Schmidt shower meteors: Mean values and standard deviations of log τ_p/ρ_m^2 for the principal showers. See table 5.

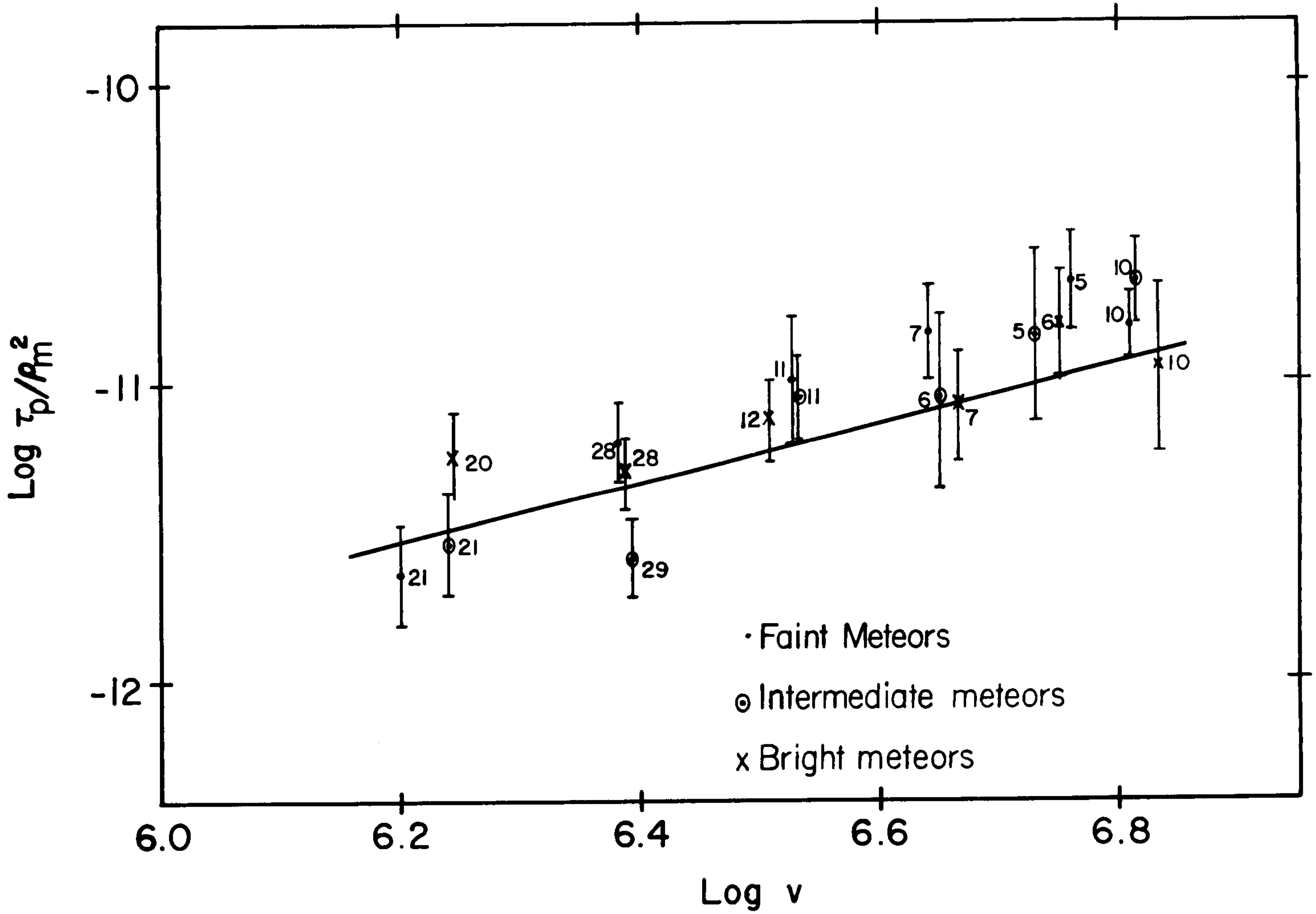

Figure 5.--Sporadic Super-Schmidt meteors: log τ_p/ρ_m^2 as a function of meteor velocity for three groups of different brightness. See table 7.

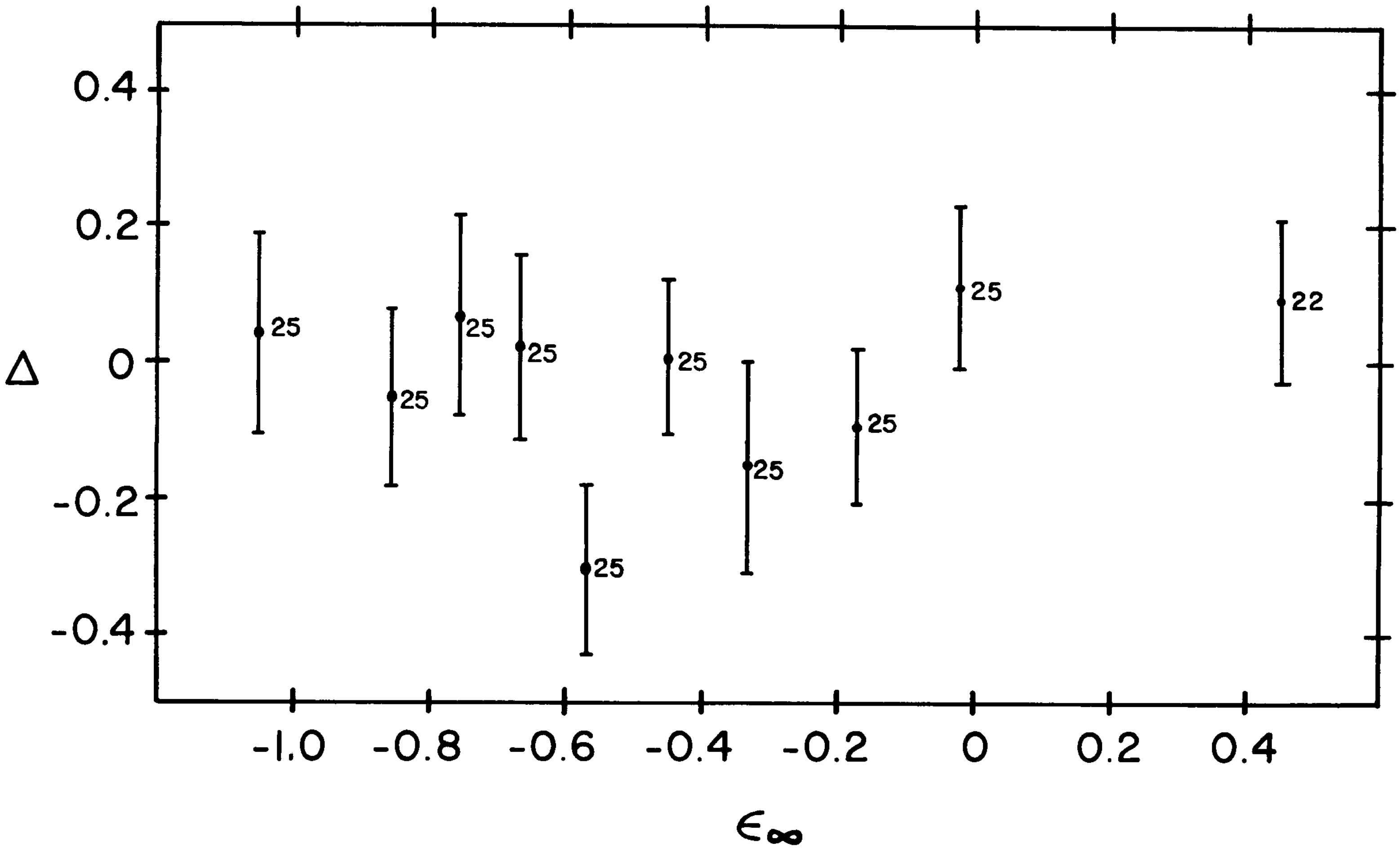

Figure 6.--Sporadic Super-Schmidt meteors: average residuals Δ for groups of meteors of different brightness ϵ_∞ , $\Delta = (\log \tau_p/\rho_m^2)_{obs} - (\log \tau_p/\rho_m^2)_{least\text{-}squares}$. See table 8.

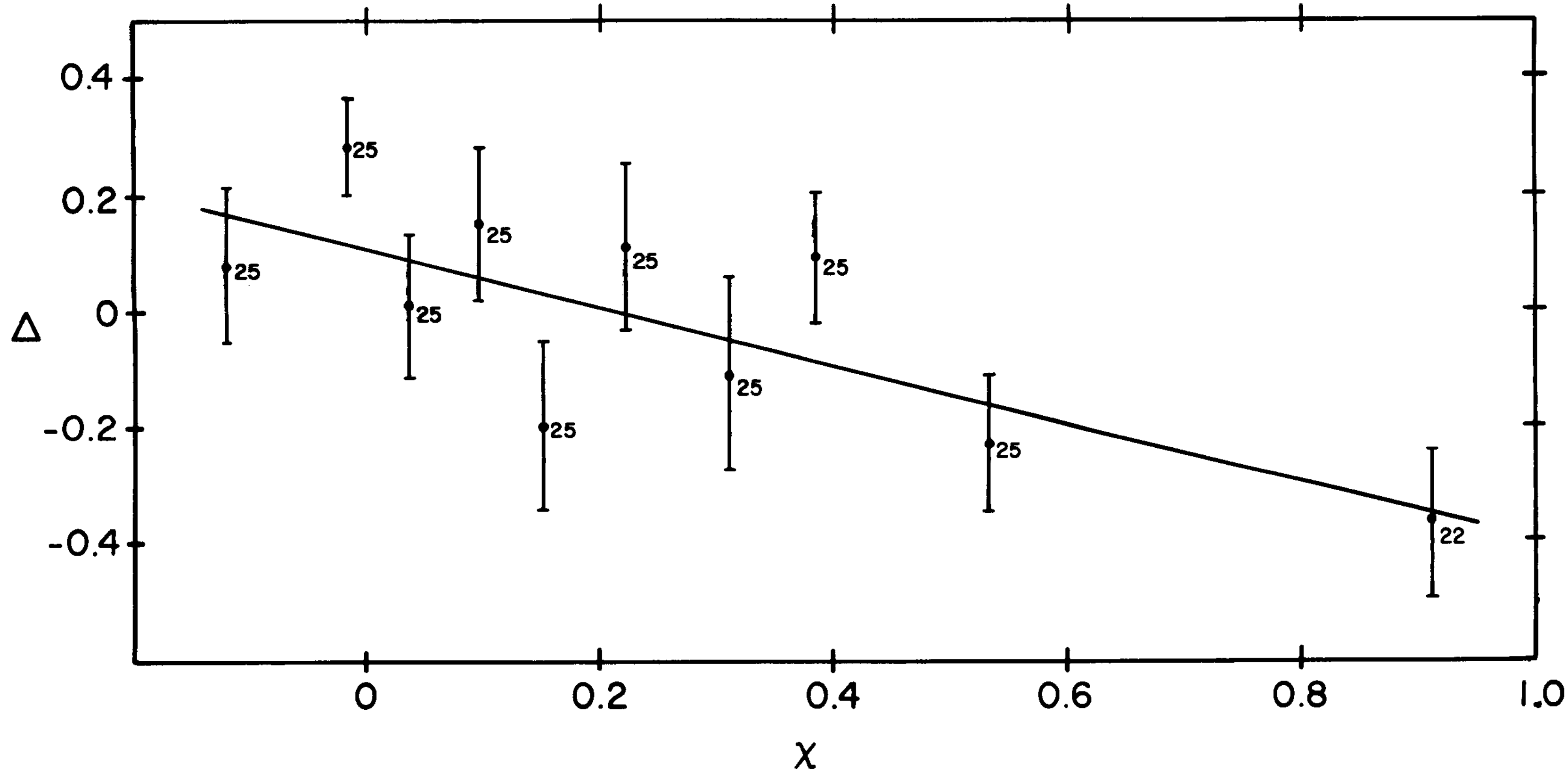

Figure 7.--Sporadic Super-Schmidt meteors: average residuals Δ for groups of meteors of different fragmentation. Straight line from a least-squares solution, $\Delta = (\log \tau_p/\rho_m^2)_{obs} - (\log \tau_p/\rho_m^2)_{least\text{-}squares}$. See table 9.

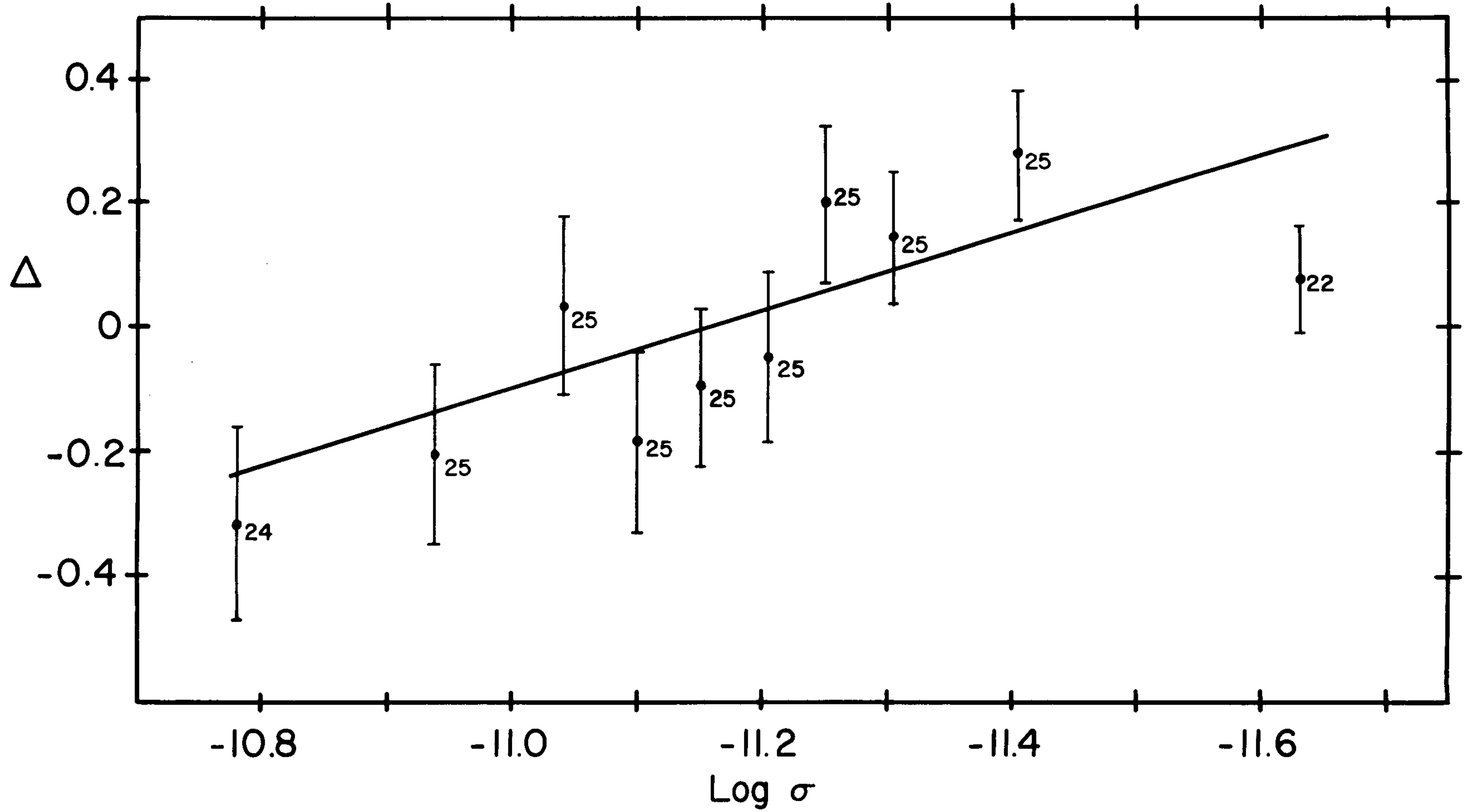

Figure 8.--Sporadic Super-Schmidt meteors: average residuals Δ vs. log σ . Straight line from a least-squares solution, $\Delta = (\log \tau_p/\rho_m^2)_{obs} - (\log \tau_p/\rho_m^2)_{least\text{-}squares}$.

Table 1

Sporadic Super-Schmidt meteors: a) Results of the general least-squares solution and mean values of the basic quantities; b) log τ_p/ρ_m^2 as a function of meteor velocity. (No correction applied for the difference in density between short-period and long-period meteors.)

a) $n = 1.52 \pm 0.15$; $\log \tau_{op}/\rho_m^2 = -20.99 \pm 0.99$; a.d = 0.53; no. of meteors = 247

$\overline{\log v} = 6.433 \pm 0.008$; $\overline{\log \tau_p/\rho_m^2} = -11.182 \pm 0.031$; $\overline{\epsilon}_\infty = -0.40 \pm 0.02$; $\overline{w} = 4.35$,

where $\epsilon_\infty = \log E_\infty$, and a.d = average deviation of individual value of $\log \tau_p/\rho_m^2$ from the least-squares solution.

b)

Velocity, v (10^5 cms^{-1})	$\overline{\log v}$	$\overline{\log \tau_p/\rho_m^2}$	S.d. of mean	$\overline{\epsilon_\infty}$	S.d. of mean	no.	$\overline{w}$
10-15	6.130	-11.898	± 0.311	-0.73	± 0.13	10	4.82
15-20	6.245	-11.377	0.095	-0.42	0.07	52	4.96
20-25	6.347	-11.397	0.105	-0.51	0.06	45	4.74
25-30	6.436	-11.227	0.104	-0.47	0.07	40	4.37
30-35	6.505	-10.996	0.101	-0.20	0.09	24	5.09
35-40	6.572	-10.753	0.217	-0.46	0.13	10	3.69
40-45	6.633	-10.730	0.160	-0.46	0.08	12	3.73
45-50	6.678	-10.806	0.150	-0.07	0.18	8	4.67
50-55	6.720	-10.544	0.230	-0.27	0.16	6	3.17
55-60	6.760	-10.465	0.139	-0.17	0.16	10	3.34
60-65	6.791	-10.887	0.198	-0.41	0.09	13	2.82
65-70	6.829	-10.934	0.164	-0.26	0.07	12	2.70
70-75	6.854	-10.534	0.454	+0.06	0.21	5	3.23

The least-squares solution for mean values gives $n = 1.51 \pm 0.15$.
Correlation coefficient between mean values of log v and $\log \tau_p/\rho_m^2 = 0.90$.

Table 2a

Sporadic Super-Schmidt meteors divided in two groups, short-period ($Q < 7$ a.u.) and long-period meteors ($Q > 7$ a.u.). Mean values of basic quantities for both groups:

a1) $Q < 7$ a.u.: no. = 155; $\overline{\log v} = 6.338 \pm 0.007$;

$$\overline{\log \tau_p/\rho_m^2} = -11.368 \pm 0.039;\quad \bar{\epsilon}_\infty = -0.49 \pm 0.02$$

$Q > 7$ a.u.: no. = 92; $\overline{\log v} = 6.621 \pm 0.013$;

$$\overline{\log \tau_p/\rho_m^2} = -10.803 \pm 0.041;\quad \bar{\epsilon}_\infty = -0.22 \pm 0.03$$

By reducing the values of $\log \overline{\tau_p/\rho_m^2}$ to equal velocity v' by successive approximations, we obtain

$$(\overline{\log \tau_p/\rho_m^2})_{\substack{Q>7\\ v=v'}} - (\overline{\log \tau_p/\rho_m^2})_{\substack{Q<7\\ v=v'}} = 0.28 \pm 0.12$$

a2) The overlapping velocity region is 20-45 km/s. For meteors having velocities between these two limits we have:

$Q < 7$ a.u.: no. = 94; $\overline{\log v} = 6.417$; $\overline{\log \tau_p/\rho_m^2} = -11.295 \pm 0.047$; $\bar{\epsilon}_\infty = -0.50 \pm 0.02$

$Q > 7$ a.u.: no. = 37; $\overline{\log v} = 6.499$; $\overline{\log \tau_p/\rho_m^2} = -10.878 \pm 0.062$; $\bar{\epsilon}_\infty = -0.26 \pm 0.05$

Then: $$(\overline{\log \tau_p/\rho_m^2})_{\substack{Q>7\\ v=v'}} - (\overline{\log \tau_p/\rho_m^2})_{\substack{Q<7\\ v=v'}} = 0.33 \pm 0.12\,.$$

From the results listed in a1) and a2), I have assumed:

$$(\overline{\log \tau_p/\rho_m^2})_{\substack{Q>7\\ v=v'}} - (\overline{\log \tau_p/\rho_m^2})_{\substack{Q<7\\ v=v'}} = 0.30.$$

a3) For low-fragmentation meteors ($|\chi| < 0.2$), in the overlapping velocity range:

$Q < 7$ a.u.: no. = 33; $\overline{\log v} = 6.462$; $\overline{\log \tau_p/\rho_m^2} = -11.167 \pm 0.082$

$Q < 7$ a.u.: no. = 18; $\overline{\log v} = 6.512$; $\overline{\log \tau_p/\rho_m^2} = -10.866 \pm 0.086$.

Then: $$(\overline{\log \tau_p/\rho_m^2})_{\substack{Q>7\\ |\chi|<0.2\\ v=v'}} - (\overline{\log \tau_p/\rho_m^2})_{\substack{Q<7\\ |\chi|<0.2\\ v=v'}} = 0.25 \pm 0.17$$

Table 2b

Sporadic Super-Schmidt meteors divided in two groups, short-period ($Q < 7$ a.u.) and long-period ($Q < 7$ a.u.): $\log \tau_p/\rho_m^2$ as a function of meteor velocity for each group.

	$Q < 7$				$Q > 7$			
Velocity, v (10^5 cms^{-1})	no.	$\overline{\log v}$	$\overline{\log \tau_p/\rho_m^2}$	S.d.of mean	no.	$\overline{\log v}$	$\overline{\log \tau_p/\rho_m^2}$	S.d.of mean
10 - 15	9	6.124	-12.065	± 0.301	1			
15 - 20	51	6.245	-11.384	0.097	1			
20 - 25	40	6.346	-11.418	0.113	5	6.353	-11.262	± 0.290
25 - 30	32	6.433	-11.324	0.124	8	6.447	-10.896	0.127
30 - 35	15	6.508	-11.081	0.119	9	6.499	-10.857	0.181
35 - 40	4	6.569	-11.090	0.387	6	6.573	-10.576	0.253
40 - 45	3	6.621	-10.616	0.161	9	6.637	-10.769	0.210
45 - 50	1				7	6.680	-10.810	0.171
50 - 55					6	6.720	-10.544	0.230
55 - 60					10	6.760	-10.465	0.139
60 - 65					13	6.791	-10.887	0.198
65 - 70					12	6.829	-10.934	0.164
70 - 75					5	6.854	-10.534	0.454

<u>Table 3</u>

Sporadic Super-Schmidt meteors: a) results of the general least-squares solution and mean values of the basic quantities; b) log τ_p/ρ_m^2 as a function of meteor velocity: mean values of log τ_p/ρ_m^2 for 6 groups of different v. (All values of log τ_p/ρ_m^2 reduced to the density of short-period meteors.)

a) n = 1.01 ± 0.15; log τ_{op}/ρ_m^2 = -17.80 ± 0.98; a.d. = 0.53; no. of meteors = 247

$\overline{\log v}$ = 6.433 ± 0.008; $\overline{\log \tau_p/\rho_m^2}$ = -11.281 ± 0.030; $\bar{\epsilon}_\infty$ = -0.40 ± 0.02; $\bar{w}$ = 4.35;

a.d. = average deviation of (the) individual value of log τ_p/ρ_m^2 from the least-squares straight line.

b)

Velocity v (10^5 cms^{-1})	$\overline{\log v}$	$\overline{\log \tau_p/\rho_m^2}$	S.d.of mean	$\bar{\epsilon}_\infty$	S.d.of mean	no.	$\bar{w}$
10 - 20	6.227	-11.469	± 0.095	-0.47	± 0.06	62	4.94
20 - 30	6.387	-11.373	0.073	-0.49	0.04	85	4.57
30 - 40	6.520	-11.073	0.089	-0.26	0.08	34	4.68
40 - 50	6.653	-11.009	0.115	-0.28	0.10	20	4.10
50 - 60	6.745	-10.794	0.118	-0.21	0.12	16	3.28
60 - 73	6.818	-11.138	0.130	-0.26	0.07	30	2.84

The least-squares solution for mean values affords: n = 0.93 ± 0.17.

Correlation coefficient between mean values of log v and log τ_p/ρ_m^2 = 0.87.

Table 4

Sporadic Super-Schmidt meteors: log τ_p/ρ_m^2 as a function of meteor velocity. (All values reduced to the density of short-period meteors.)

Velocity, v (10^5 cms^{-1})	$\overline{\log v}$	$\overline{\log \tau_p/\rho_m^2}$	S.d.of mean	$\overline{\epsilon_\infty}$	S.d.of mean	no.	$\overline{w}$
10 - 15	6.130	-11.931	± 0.296	-0.73	± 0.13	10	4.82
15 - 20	6.245	-11.383	0.095	-0.42	0.07	52	4.96
20 - 25	6.347	-11.438	0.105	-0.51	0.06	45	4.74
25 - 30	6.436	-11.295	0.101	-0.47	0.07	40	4.37
30 - 35	6.505	-11.110	0.098	-0.20	0.09	24	5.09
35 - 40	6.572	-10.949	0.204	-0.46	0.13	10	3.69
40 - 45	6.633	-10.954	0.169	-0.46	0.08	12	3.73
45 - 50	6.678	-11.076	0.155	-0.07	0.18	8	4.67
50 - 55	6.720	-10.844	0.230	-0.27	0.16	6	3.17
55 - 60	6.760	-10.765	0.139	-0.17	0.16	10	3.34
60 - 65	6.791	-11.187	0.198	-0.41	0.09	13	2.82
65 - 70	6.829	-11.234	0.164	-0.26	0.07	12	2.70
70 - 75	6.854	-10.834	0.454	+0.06	0.21	5	3.23

The least-squares solution for mean values affords: $n = 1.00 \pm 0.15$.
Correlation coefficient between mean values of log v and log $\tau_p/\rho_m^2 = 0.81$*.

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*The decrease of the correlation coefficient with respect to the uncorrected data of table 1 is caused by the decrease of the value of n . Such a decrease does not mean a worse correlation.

Table 5

Super-Schmidt shower-meteors: basic values and relative density of each shower.

Shower	no.	$\overline{\log v} \pm$ s.d.	$\overline{\log \tau_p/\rho_m^2} \pm$ s.d.	$\bar{\varepsilon}_\infty \pm$ s.d.	$\Delta_\rho \pm$ s.d.	$\overline{\rho_{sh}}/\overline{\rho_{sp}} \pm$ s.d.	$\bar{Q}$
Geminids	20	6.556 ± 0.001	-12.357 ± 0.161	-0.06 ± 0.14	-1.20 + 0.23	4.0 ± 1	2.6
Southern Taurids	18	6.455 ± 0.006	-11.137 ± 0.079	-0.42 ± 0.10	0.12 ± 0.12	0.9 ± 0.1	3.5
α - Capricornids	13	6.400 ± 0.009	-10.733 ● 0.167	-0.55 ± 0.13	0.58 ± 0.22	0.5 ± 0.1	5.3
δ - Aquarids	10	6.624 ± 0.003	-11.106 ± 0.176	-0.59 ± 0.11	-0.02 ± 0.26	1.0 ± 0.3	5.5
Quadrantids	9	6.630 ± 0.002	-10.700 ± 0.184	-0.62 ± 0.09	0.38 ± 0.27	0.65 ± 0.2	5.2
Perseids	9	6.778 ± 0.001	-11.099 ± 0.125	0.31 ± 0.21	-0.17 ± 0.24	$1.2 \pm^{0.4}_{0.3}$	45
Orionids	8	6.831 ± 0.003	-10.428 ± 0.217	-0.09 ± 0.10	0.45 ± 0.35	$0.6 \pm^{0.3}_{0.2}$	30
Southern ι - Aquarids	6	6.524 ± 0.014	-11.296 ± 0.133	-0.75 ± 0.12	-0.11 ± 0.19	1.1 ± 0.3	4.2
Northern Taurids	5	6.484 ± 0.007	-11.201 ± 0.184	0.08 ± 0.20	0.03 ± 0.23	1.0 ± 0.3	4.2
κ - Cygnids	4	6.358 ± 0.008	-10.909 ± 0.170	-0.65 ± 0.05	0.45 ± 0.23	0.6 ± 0.2	5
Lyrids	3	6.688 ± 0.010	-11.201 ± 0.281	-0.12 ± 0.12	-0.18 ± 0.38	$1.2 \pm^{0.7}_{0.5}$	55
σ - Hydrids	3	6.771 ± 0.004	-11.181 ± 0.328	-0.28 ± 0.08	-0.24 ± 0.43	$1.3 \pm^{0.8}_{0.5}$	60
Draconids*	2	6.235 ± 0.001	- 6.001 ± 0.327	-0.28 ± 0.03	5.48 ± 0.4	$(2\pm1) \cdot 10^{-3}$	5.6
Virginids	2	6.461 ± 0.051	-12.310 ± 0.220	-0.20 ± 0.29	-1.06 ± 0.27	3.5 ± 1	5

$$\Delta_\rho = (\overline{\log \tau_p/\rho_m^2})_{shower} - (\overline{\log \tau_p/\rho_m^2})_{\substack{sporadic,\ Q<7 \\ v = v_{shower}}} = 2 \log \frac{\rho_{sp}}{\rho_{sh}} .$$

ρ_{sh} = density of the meteoroids of the shower

ρ_{sp} = density of the sporadic meteoroids with $Q < 7$.

Q = average aphelion distance in a.u.: from Jacchia (1963) and Whipple and Jacchia (1961) .

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*Note: The fragmentation index χ for these 2 meteors is very large (respectively 1.32 and 2.46), so that the values of $\overline{\log \tau_p/\rho_m^2}$ and of ρ_{sh}/ρ_{sp} are practically meaningless.

Table 6

Sporadic Super-Schmidt meteors with fragmentation index $|\chi| < 0.2$: a) basic results; b) $\log \tau_p/\rho_m^2$ as a function of meteor velocity. (All values of $\log \tau_p/\rho_m^2$ reduced to the density of short-period meteors.)

a) $n = 1.24 \pm 0.22$; $\log \tau_{op}/\rho_m^2 = -19.18 \pm 1.41$; a.d = 0.51; no. of meteors = 125;

$\overline{\log v} = 6.421 \pm 0.011$; $\overline{\log \tau_p/\rho_m^2} = -11.214 \pm 0.040$; $\overline{\epsilon_\infty} = -0.33 \pm 0.028$; $\bar{w} = 4.74$.

b)

Velocity, v (10^5 cms^{-1})	$\overline{\log v}$	$\overline{\log \tau_p/\rho_m^2}$	S.d.of mean	$\overline{\epsilon_\infty}$	S.d.of mean	no.	$\bar{w}$
10 - 20	6.220	-11.360	± 0.130	-0.37	± 0.09	30	5.66
20 - 30	6.394	-11.318	0.094	-0.46	0.06	49	4.69
30 - 40	6.522	-11.001	0.131	-0.22	0.10	20	4.77
40 - 50	6.665	-11.229	0.148	-0.02	0.16	8	4.95
50 - 60	6.740	-10.742	0.182	-0.04	0.15	7	3.67
60 - 73	6.819	-10.680	0.230	-0.17	0.12	11	2.87

Table 7a

Sporadic Super-Schmidt meteors: mean values of basic data for three groups, equally populated, of different brightness.

	Faintest	Intermediate	Brightest meteors
no.	83	82	82
$\overline{\epsilon_\infty}$	-0.86 ± 0.01	-0.51 ± 0.01	0.07 ± 0.02
$\overline{\log v}$	6.355 ± 0.012	6.452 ± 0.013	6.482 ± 0.015
$\overline{\log \tau_p/\rho_m^2}$	-11.341 ± 0.056	-11.362 ± 0.050	-11.165 ± 0.047
$(\overline{\log \tau_p/\rho_m^2})_{\log v = 6.40}$	-11.30 ± 0.07	-11.41 ± 0.06	-11.25 ± 0.06
$\overline{\chi}$	0.24 ± 0.05	0.29 ± 0.04	0.13 ± 0.03
$\overline{\log \sigma}$	-11.15 ± 0.04	-11.17 ± 0.04	-11.24 ± 0.03
$\overline{s}$	-0.06 ± 0.02	-0.13 ± 0.03	-0.25 ± 0.04
$(\overline{\log \tau_p/\rho_m^2})^*_{\log v = 6.40}$ $\chi = 0.22$ $\log \sigma = -11.19$ $s = -0.15$	-11.35 ± 0.14	-11.40 ± 0.13	-11.31 ± 0.11

*The reduction of the $\overline{\log \tau_p/\rho_m^2}$ to standard values of χ, $\log \sigma$, and s is done by means of the correlations discussed in sections 3b and 3d.

Table 7b

Sporadic Super-Schmidt meteors: $\log \tau_p/\rho_m^2$ as a function of meteoric velocity for three groups of different brightness and equal speed distribution

	FAINTEST				INTERMEDIATE				BRIGHTEST METEORS			
Velocity, v (10^5 cms^{-1})	no.	$\overline{\log v}$	$\overline{\log \tau_p/\rho_m^2}$ ± S.d.	$\bar{\epsilon}_\infty$ ± S.d.	no.	$\overline{\log v}$	$\overline{\log \tau_p/\rho_m^2}$ ± S.d.	$\bar{\epsilon}_\infty$ ± S.d.	no.	$\overline{\log v}$	$\overline{\log \tau_p/\rho_m^2}$ ± S.d.	$\bar{\epsilon}_\infty$ ± S.d.
10-20	21	6.196	-11.65 ± 0.17	-0.94 ± 0.03	21	6.239	-11.55 ± 0.17	-0.62 ± 0.03	20	6.242	-11.25 ± 0.15	0.06 ± 0.07
20-30	28	6.382	-11.21 ± 0.13	-0.90 ± 0.03	29	6.393	-11.60 ± 0.13	-0.61 ± 0.02	28	6.386	-11.31 ± 0.12	-0.10 ± 0.07
30-40	11	6.528	-11.00 ± 0.21	-0.80 ± 0.04	11	6.529	-11.05 ± 0.14	-0.40 ± 0.05	12	6.509	-11.13 ± 0.14	0.18 ± 0.06
40-50	7	6.636	-10.84 ± 0.16	-0.76 ± 0.05	6	6.652	-11.07 ± 0.29	-0.35 ± 0.04	7	6.666	-11.08 ± 0.18	0.09 ± 0.13
50-60	5	6.758	-10.67 ± 0.16	-0.75 ± 0.06	5	6.729	-10.85 ± 0.29	-0.37 ± 0.06	6	6.750	-10.82 ± 0.18	0.22 ± 0.12
60-73	10	6.807	-10.82 ± 0.11	-0.60 ± 0.05	10	6.811	-11.67 ± 0.14	-0.34 ± 0.02	10	6.832	-10.96 ± 0.28	0.08 ± 0.10
Global	82	6.416	-11.23 ± 0.08	-0.86 ± 0.02	82	6.435	-11.44 ± 0.08	-0.53 ± 0.02	83	6.444	-11.19 ± 0.07	0.04 ± 0.04
		n = 1.59 ± 0.25				n = 0.95 ± 0.27				n = 0.57 ± 0.24		

Table 8

Sporadic Super-Schmidt meteors: residuals Δ as a function of integrated brightness .

$\overline{\epsilon_\infty}$	S.d.of mean	$\overline{\Delta}$	S.d.of mean	no.
-1.049	± 0.025	0.046	± 0.147	25
-0.854	0.007	-0.051	0.130	25
-0.756	0.005	0.070	0.146	25
-0.664	0.004	0.022	0.137	25
-0.568	0.007	-0.302	0.126	25
-0.447	0.005	0.009	0.118	25
-0.329	0.008	-0.153	0.156	25
-0.169	0.008	0.093	0.114	25
+0.019	0.013	0.111	0.124	25
+0.454	0.045	0.093	0.121	22

$$\Delta = \left(\log \tau_p/\rho_m^2\right)_{obs.} - \left(\log \tau_p/\rho_m^2\right)_{least\text{-}squares.}$$

<u>Table 9</u>

Sporadic Super-Schmidt meteors: residuals Δ as a function of the fragmentation index χ.

$\bar{\chi}$	S.d.of mean	$\bar{\Delta}$	S.d.of mean	no.
-0.122	± 0.014	0.082	± 0.132	25
-0.018	0.003	0.281	0.082	25
0.036	0.004	0.016	0.120	25
0.096	0.004	0.153	0.132	25
0.149	0.003	-0.197	0.146	25
0.222	0.005	0.116	0.140	25
0.309	0.005	-0.111	0.164	25
0.383	0.005	0.091	0.112	25
0.532	0.015	-0.232	0.118	25
0.909	0.052	-0.368	0.124	22

Correlation coefficient between mean values of Δ and χ = -0.73.

Table 10

Sporadic Super-Schmidt meteors: residuals Δ as a function of log σ.

$\overline{\log \sigma}$	S.d.of mean	$\bar{\Delta}$	S.d.of mean	no.
-10.785	± 0.016	-0.317	± 0.155	24
-10.937	0.007	-0.207	0.146	25
-11.040	0.005	0.035	0.145	25
-11.099	0.003	-0.184	0.146	25
-11.150	0.003	-0.094	0.128	25
-11.204	0.003	-0.051	0.138	25
-11.251	0.002	0.200	0.125	25
-11.304	0.004	0.141	0.108	25
-11.406	0.007	0.287	0.105	25
-11.633	0.032	0.078	0.086	22

Correlation coefficient between mean values of Δ and log σ = -0.75.

Table 11

Values of $\log \tau_p/\rho_m^2$ computed by single decelerations at different heights for each of several sporadic Super-Schmidt meteors.

Trail no.	Height (km)	$\log \rho_a (gcm^{-3})$	s	Plate	$\log \tau_p/\rho_m^2$	p
9030	90.01	-8.50	-0.22	ST	-11.42	7.7
	89.64	-8.47	-0.18	SL	-11.31	7.7
$v = 23.7\ kms^{-1}$	87.78	-8.33	0.04	SL	-11.28	6.9
	86.86	-8.25	0.38	ST	-11.43	6.1
$\chi = 0.00$	82.24	-7.88	0.80	SL	-11.38	3.5
	82.23	-7.88	0.80	ST	-11.27	3.4
5572	87.35	-8.29	-0.56	ST	-11.10	8.5
	87.22	-8.28	-0.54	SS	-11.16	7.8
$v = 16.4\ kms^{-1}$	82.86	-7.93	0.07	SS	-11.21	5.6
	82.75	-7.92	0.08	ST	-11.34	5.3
$\chi = 0.00$	79.88	-7.69	0.71	ST	-11.02	3.5
	79.80	-7.69	0.75	SS	-11.21	3.4
12361	93.13	-8.76	-0.54	SK	-10.93	8.8
	92.53	-8.71	-0.42	SL	-10.80	8.5
$v = 22.1\ kms^{-1}$	91.43	-8.61	-0.19	SK	-10.80	7.8
	91.12	-8.59	-0.13	SL	-10.80	7.6
$\chi = 0.01$	90.40	-8.53	0.03	SK	-10.91	6.9
	90.22	-8.52	0.07	SL	-10.87	6.7
	88.08	-8.35	0.90	SL	-10.99	3.1
	87.85	-8.33	1.07	SK	-10.80	2.7
11973	98.83	-9.22	-0.53	ST	-11.02	8.6
	98.30	-9.18	-0.47	SL	-11.20	8.4
$v = 44.7\ kms^{-1}$	97.38	-9.11	-0.38	ST	-10.95	8.4
	96.84	-9.07	-0.32	SL	-11.12	8.1
$\chi = -0.01$	91.04	-8.58	0.53	SL	-11.20	3.6
	90.80	-8.56	0.57	ST	-10.98	3.7
5532	87.03	-8.26	-0.39	ST	-11.85	8.0
	86.68	-8.23	-0.35	SL	-11.70	7.9
$v = 22.5\ kms^{-1}$	85.07	-8.10	-0.15	ST	-11.76	7.5
	84.87	-8.09	-0.12	SL	-11.68	7.4
$\chi = 0.01$	80.46	-7.74	0.67	SL	-11.76	3.7
7161	105.78	-9.73	-0.85	SL	-10.80	9.0
	105.25	-9.69	-0.79	ST	-10.74	9.1
$v = 48.8\ kms^{-1}$	102.74	-9.51	-0.51	ST	-10.71	8.7
	102.31	-9.47	-0.46	SL	-10.68	8.6
$\chi = -0.01$	101.57	-9.42	-0.38	ST	-10.50	7.9
	100.90	-9.37	-0.31	SL	-10.63	8.2
	96.64	-9.04	0.17	ST	-10.84	6.2
	94.79	-8.90	0.44	SL	-10.74	5.0

Table 11 (continued)

Trail no.	Height (km)	log ρ_a (gcm^{-3})	s	Plate	log τ_p/ρ_m^2	p	$(\log \tau_p/\rho_m^2)_{s=0}$
5195	96.52	-9.02	-0.42	SL	-10.64	7.0	
	95.97	-8.99	-0.34	ST	-10.70	7.2	
$v = 28.4\ kms^{-1}$	88.73	-8.40	0.68	SL	-10.64	2.2	
	88.63	-8.39	0.69	ST	-10.53	2.0	
$\chi = 0.02$							
8640	106.14	-9.75	-0.33	SK	-10.13	8.2	
	105.65	-9.72	-0.28	SL	-10.33	7.8	
$v = 55.4\ kms^{-1}$	101.44	-9.41	0.31	SK	-10.17	5.1	
	101.05	-9.38	0.35	SL	-10.29	4.4	
$\chi = 0.02$	99.23	-9.24	0.80	SK	-10.18	3.3	
	98.88	-9.22	0.88	SL	-10.06	2.8	
7216	98.13	-9.18	-0.94	ST	-11.11	9.4	-10.94
	97.02	-9.07	-0.80	SL	-10.95	9.3	-10.81
$v = 31.4\ kms^{-1}$	94.70	-8.89	-0.54	ST	-10.86	8.8	-10.76
	93.93	-8.83	-0.45	SL	-10.80	8.6	-10.72
$\chi = 0.06$	91.35	-8.61	-0.17	SL	-10.74	7.7	-10.71
	90.31	-8.53	0.05	ST	-10.75	7.2	-10.74
	86.47	-8.22	0.68	ST	-10.77	4.0	-10.89
	85.31	-8.13	0.93	SL	-10.69	3.1	-10.86
4464	94.08	-8.83	-1.42	ST	-11.28	8.1	-10.98
	93.43	-8.78	-1.34	SS	-11.04	8.8	-10.76
$v = 20.7\ kms^{-1}$	85.45	-8.14	-0.51	ST	-10.88	8.4	-10.78
	84.24	-8.04	-0.40	SS	-11.02	8.3	-10.94
$\chi = 0.07$	80.72	-7.76	-0.03	ST	-11.20	6.5	-11.20
	79.04	-7.64	0.16	SS	-10.94	6.1	-10.97
	77.39	-7.52	0.39	ST	-10.70	4.6	-10.78
	75.48	-7.40	0.99	SS	-10.55	2.7	-10.76
	74.76	-7.35	1.42	ST	-10.51	1.4	-10.81
7946	77.49	-7.53	-1.05	SL	-14.00	9.4	-13.75
	76.44	-7.46	-0.94	SK	-13.87	4.9	-13.64
$v = 17.8\ kms^{-1}$	65.78	-6.72	-0.22	SL	-14.20	7.9	-14.15
	65.69	-6.72	-0.21	SK	-14.21	7.8	-14.16
$\chi = 0.08$	59.40	-6.48	0.20	SK	-14.07	6.2	-14.12
	58.94	-6.46	0.22	SL	-14.01	6.1	-14.06
	55.56	-6.28	0.49	SK	-13.80	4.7	-13.92
	55.21	-6.26	0.52	SL	-13.72	4.8	-13.84
	51.51	-6.07	1.13	SK	-13.12	2.4	-13.39
	51.05	-6.04	1.28	SL	-13.01	2.1	-13.32

Table 11 (continued)

Trail no.	Height (km)	log ρ_a (gcm^{-3})	s	Plate	log τ_p/ρ_m^2	p	$(\log \tau_p/\rho_m^2)_{s=0}$
6949	91.68	-8.64	-0.87	SL	-12.26	9.3	-11.95
	91.10	-8.59	-0.81	ST	-12.43	9.2	-12.14
$v = 26.7$ kms^{-1}	88.63	-8.39	-0.59	SL	-12.12	8.9	-11.91
	88.40	-8.37	-0.57	ST	-12.19	8.9	-11.98
$X = 0.12$	84.00	-8.02	-0.04	SL	-12.04	7.1	-12.03
	83.50	-7.98	0.03	ST	-12.15	6.6	-12.16
	79.92	-7.69	0.59	SL	-11.83	4.2	-12.04
	79.76	-7.68	0.62	ST	-11.67	4.0	-11.89
12577	94.62	-8.88	-0.60	SL	-11.40	8.4	-11.04
	94.47	-8.87	-0.59	SK	-11.42	8.3	-11.06
$v = 29.2$ kms^{-1}	91.92	-8.66	-0.34	SL	-11.17	8.3	-10.97
	91.63	-8.64	-0.32	SK	-11.22	8.2	-11.03
$X = 0.20$	89.70	-8.48	-0.11	SK	-11.07	7.3	-11.01
	89.54	-8.47	-0.08	SL	-11.14	6.8	-11.09
	85.58	-8.15	0.53	SK	-10.81	4.3	-11.13
	85.42	-8.13	0.56	SL	-10.56	4.1	-10.90
4702	93.67	-8.81	-0.75	SS	-13.03	2.1	-10.85
	89.53	-8.46	-0.11	SS	-11.04	5.5	-10.72
$v = 31.5$ kms^{-1}	89.12	-8.43	-0.07	KB	-10.58	4.8	-10.38
$X = 0.97$	85.44	-8.14	0.51	SS	- 9.39	1.3	-10.87
10414	88.46	-8.38	-1.64	SL	-11.18	9.7	-11.67
	86.62	-8.23	-1.33	SK	-11.25	7.9	-11.65
$v = 15.6$ kms^{-1}	77.04	-7.50	-0.32	SL	-11.52	8.1	-11.62
	76.47	-7.46	-0.27	SK	-11.52	7.8	-11.60
$X = -0.10$	69.39	-7.03	0.39	SL	-11.87	5.4	-11.75
	69.26	-7.02	0.41	SK	-11.90	5.1	-11.78
	63.80	-6.72	1.19	SL	-11.90	2.5	-11.54
	63.58	-6.71	1.25	SK	-11.90	2.3	-11.53

Table 12

Sporadic small-camera meteors: a) Basic results; b) log τ_p/ρ_m^2 as a function of meteor velocity.

a)	no.	$\overline{\log v}$ ± p.e.	$\overline{\log \tau_p/\rho_m^2}$ ± p.e.	$\bar{\epsilon}_\infty$ ± p.e.	$\bar{w}$
All together	44	6.389 ± 0.019	-11.630 ± 0.084	1.43 ± 0.08	3.66
Meteors with Q < 7	30	6.324 ± 0.017	-11.779 ± 0.096	1.47 ± 0.09	4.03
Meteors with Q > 7	13	6.584 ± 0.034	-11.264 ± 0.140	1.32 ± 0.16	2.88
Meteors with ϵ_∞ > 1	26	6.378 ± 0.028	-11.868 ± 0.101	1.82 ± 0.08	4.18
Meteors with ϵ_∞ < 1	18	6.413 ± 0.022	-11.132 ± 0.115	0.63 ± 0.07	2.90

Least-squares solution for all meteors: n = 1.15 ± 0.5 (no correction applied for the difference of density between short-period and long-period meteors).

Least-squares solution for all meteors, after reduction of all data to the density of short-period meteors: n = 0.90 ± 0.5.

b)

Velocity, v (10^5 cms^{-1})	no.	$\overline{\log v}$	$\overline{\log \tau_p/\rho_m^2}$	S.d.of mean	$\overline{(\log \tau_p/\rho_m^2)}_{corr.}$	$\bar{\epsilon}_\infty$ ± S.d.	$\bar{w}$
10 - 20	11	6.191	-12.105	0.226	-12.105	1.2 ± 0.2	4.45
20 - 27	15	6.354	-11.310	0.204	-11.359	1.6 ± 0.2	3.61
27 - 40	10	6.506	-11.652	0.206	-11.708	1.5 ± 0.3	3.67
40 - 70	8	6.734	-11.305	0.341	-11.540	1.4 ± 0.2	2.64

$\overline{(\log \tau_p/\rho_m^2)}_{corr.}$ are the mean values of log τ_p/ρ_m^2 after reduction of all data to the density of short-period meteors.